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# PARTIAL SELECTIONS OF DISCRETE FOREST ALTERNATIVES: ECONOMIC IMPLICATIONS

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# PARTIAL SELECTIONS OF DISCRETE FOREST ALTERNATIVES: ECONOMIC IMPLICATIONS

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Final Report

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Methodology to Balance Adequacy of
Minimum Protective Standards for Wildlife
and Fish and the Implied Opportunity Costs
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ABSTRACT

Renewable resource decision making problems involving a continuous range of production possibilities can be represented by an initial set of optimal alternatives. Each alternative is associated with a minimum cost as determined by the evaluation process.

A single alternative, however, may not be acceptable to the decision maker. Partial selections, fractional combinations of optimal alternatives, can be useful in generating additional alternatives without incurring the expense of complete re-evaluation. Output quantities and cost are derived as weighted averages of the values for optimal alternatives being combined.

Theoretical considerations suggest that cost values predicted in this manner are minimum only for a constant marginal cost production system. For increasing marginal cost functions, partial selection alternatives must be feasible but suboptimal. Determining the degree of suboptimality requires knowledge of the form and possible interactions of marginal cost functions.

The purpose of this study was to investigate the validity of hypotheses developed from theoretical considerations concerning the error in cost predictions for partial

selection alternatives using realistic forest planning methods and data. The feasibility hypothesis was supported by 14 of 15 partial selections tested. This is important, as suboptimal alternatives are at least operable (feasible).

# CHAPTER ONE

## INTRODUCTION

Renewable resource decision making problems often involve so large a spectrum of possible choices that explicit enumeration of all alternatives is not feasible due to restrictions on cost or time, or lack of knowledge about the production relationships involved. For such situations an initial set of alternatives reflecting the range of choices available may be used as a practical representation of the problem. These alternatives might be generated, for example, through use of an optimization model based on economic criteria.

If the decision maker is not satisfied with any one alternative from the initial set, additional alternatives must be generated (evaluated) for consideration. Because the results of the evaluation process cannot be predicted, there is no guarantee that one of the new alternatives will be acceptable. Numerous cycles of this process may be required before a preferred alternative is generated. This may not be practical in situations where cost or time is limited.

## The General Case of Partial Selections

Partial selections (USDA Forest Service 1982) of alternatives, taking fractional combinations of alternatives, would be a quick and inexpensive method of filling in the informational gap between inital alternatives. The set of initial alternatives provides extreme points in making predictions pertaining to alternatives generated after the initial cycle. Thus, the sometimes costly evaluation process is avoided after the initial set of optimal alternatives is generated.

For example, consider the situation shown in Figure 1. Points 1, 2 and 3 represent output combinations for three initial alternatives. Associated with each of the three alternatives is an output combination and minimum cost as determined by the evaluation process. The output quantities could represent production by a multi-output firm or outcomes of alternative public policies. For the situations shown, alternatives 1, 2 and 3 lie on successively higher production possibilities or budget curves. It is important to note that the only things known about the production possibilities curves are the location of points 1, 2 and 3. If the production relationships were fully known, a continuous set of alternatives could be analyzed and the problem being addressed would not exist.

Partial selections between alternatives 1 and 2 permits the decision maker to choose any output combination

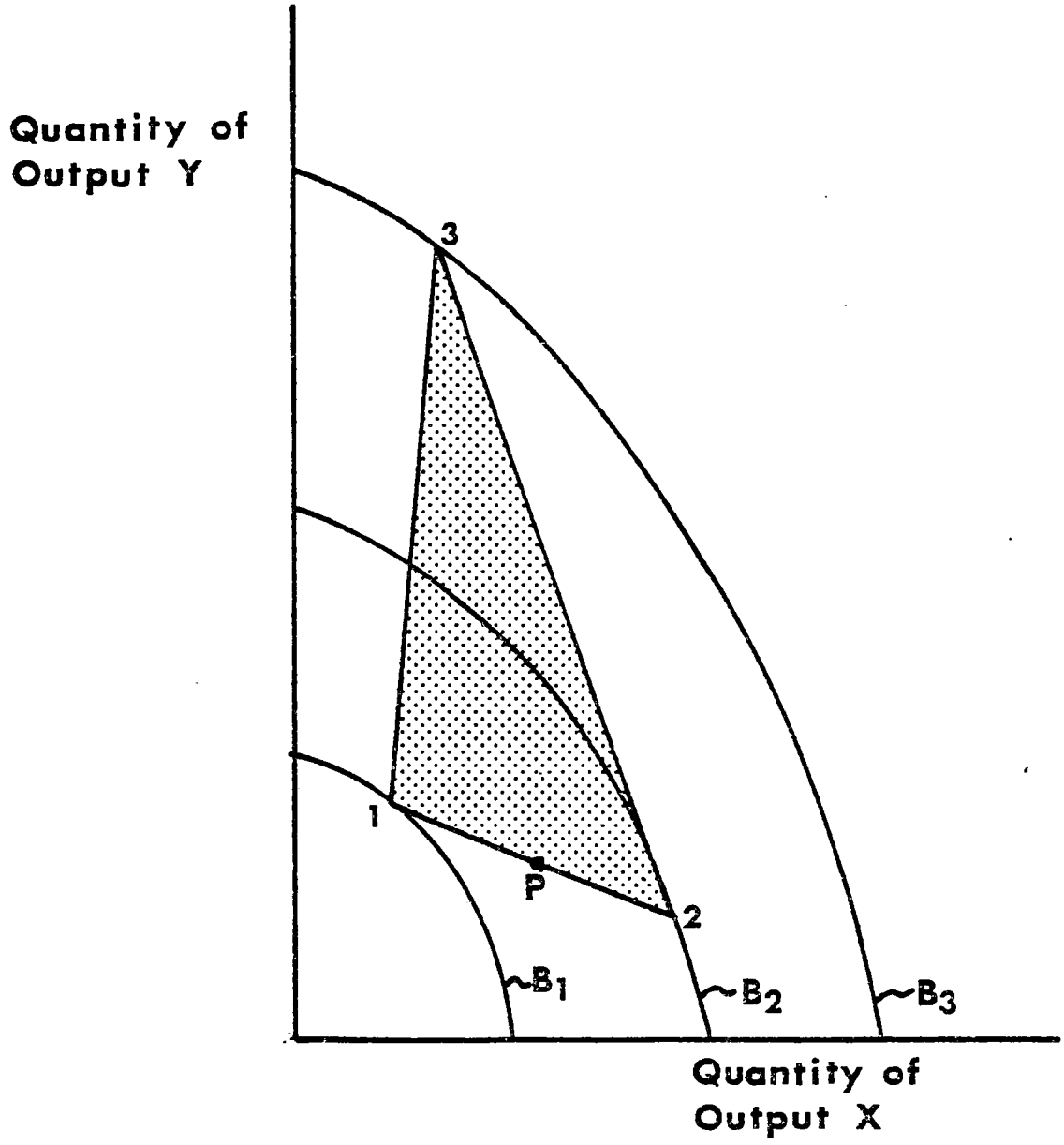


Figure 1. Region of possible output combinations for partial selections of three optimal alternatives.

along the line segment between points 1 and 2 in the figure. For example, if the decision maker wishes to consider an equal blend of the two alternatives, the new partial selection alternative will have an output combination represented by P, the midpoint of the segment, and calculated as the simple average of the original output quantities.

The most obvious method of predicting the cost of the new partial selection output combination would be to average the cost values of the original alternatives. Mathematically,

$$B_P = 0.5B_1 + 0.5B_2$$

where $B_P$ is the predicted cost of the partial selection output combination. For the more general case of any fractional combination of the two alternatives, the cost is predicted as:

$$B_P = sB_1 + (1 - s)B_2$$

where s is the proportion of the first (lower cost) alternative selected.

Partial selections between any or all of the three alternatives expands the possible choices available to the decision maker to include any output combination in the shaded triangle. However, while the initial alternatives are associated with cost values that are known to be minimum from the evaluation process, least cost output combinations

cannot be guaranteed for partial selection alternatives. The partial selection output combination could in actuality be unattainable at the predicted cost, and the alternative would be infeasible. If the output quantities could be produced at a lower cost than predicted, the partial selection alternative is suboptimal.

## Partial Selections in Forest Planning

In the National Forest planning process, a set of alternatives reflecting varying output emphases is used as a representation of the production possibilities for a given forest. The number of alternatives in the set is restricted by the nature of the evaluation process. Generation of each alternative requires a computer run of a linear programming (LP) model such as FORPLAN (Johnson, Jones and Kent 1980). The LP model assigns acreage to treatments such that the resulting land allocation optimizes the given objective function without violating constraint conditions. By choice of objective function and constraint conditions, each run generates an alternative with a unique set of outputs, benefits, cost and land allocation. The evaluation process also requires analysis of socioeconomic and environmental impacts for each alternative using computer methods as well as professional judgement.

Because only a finite set of alternatives can be generated, no single alternative in the set analyzed may be totally acceptable. In these instances, partial selections

could be a useful technique in avoiding the entire evaluation process in generating new alternatives. The decision maker could use the minimum cost values for the optimal alternatives in predicting the cost of new partial selection output combinations.

## Partial Selections in Multilevel Forest Planning Models

A problem found in National Forest land management planning is coordination of plans at forest, regional and national levels. The multilevel model described by Wong (1980) for national planning of forest allocation and production and the National Assessment Multiresource Model (NAMM) proposed by Hof (1983) for the 1990 RPA Assessment are examples of computer models that are designed to direct selection of individual forest alternatives to meet regional and national objectives.

A simplified version of the NAMM model (Hof 1983) is presented in Figure 2 for a forest system of two regions, each containing two National Forests. At the local level, a detailed analysis of production possibilities using an LP forest alternative generator such as FORPLAN (Johnson, Jones and Kent 1980) yields a set of proposed (optimal) alternatives with associated outputs, benefits and cost. This process is consistent with the present planning process at the forest level.

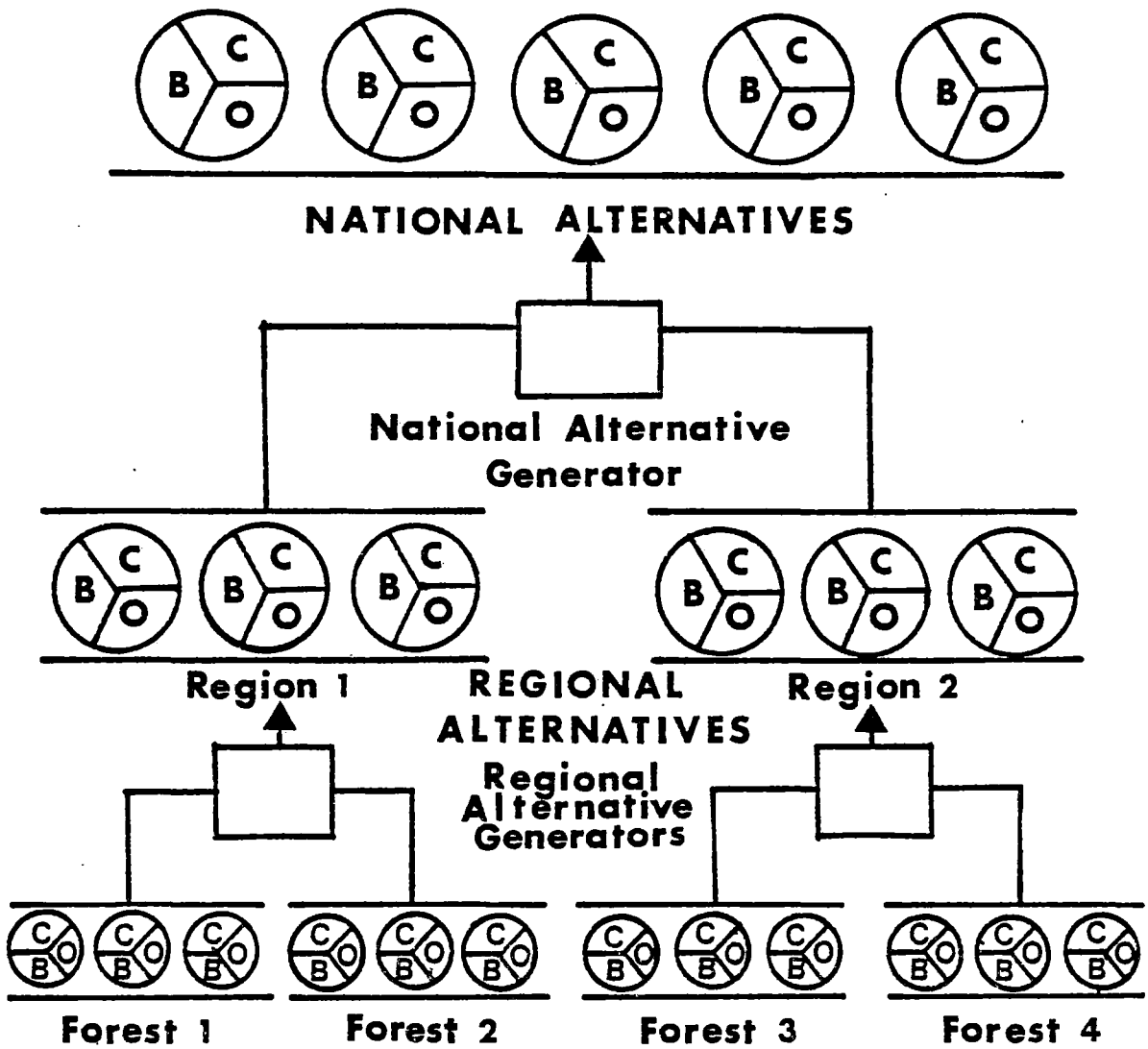


Figure 2. A simplified multilevel model for forest planning.

Selection of alternatives for individual forests is performed by the regional alternative generator LP model. Objective function and constraint conditions, including output requirements, can be varied to generate a wide range of production possibilities for the set of alternatives for a given region. National alternatives are generated in the same manner from the regional alternatives by the national alternative generator LP model.

The individual forest alternatives that serve as choice variables for the regional LP model and the resulting regional alternatives that serve as choice variables for the national model can be treated as continuous or binary (zero, one) variables (Bradley, Hax and Magnanti 1977). If alternatives are constrained to be binary choice variables, an alternative is either chosen by the higher level model (assigned a value of one) or not chosen (zero value). Each regional alternative would then be the combination of individual forest alternatives that optimize the regional objective function without violating constraint capacities. Likewise, national alternatives would be a summation of individual regional alternatives chosen by the national LP model.

Defining the choice variables as continuous would allow them to take on any value from zero to one. Thus the regional optimization model could choose a single alternative for a given forest or a fractional combination of

alternatives. When the latter occurs, a partial selection (USDA Forest Service 1982) has been made by the higher level model. Partial selections imply that either no feasible binary combination of the original alternatives exists to meet constraint requirements, or that improvement of the objective function can be achieved by choosing a fractional combination of alternatives (Hof, Joyce, Alward and Hoekstra 1983).

The continuous variable formulation of the model provides a vast increase in possible solutions available to the higher level model. Since binary solutions are still available to the model, the objective function value for the continuous variable formulation can be no worse in terms of optimization than for the binary model. In addition, infeasibilities may be eliminated in certain instances, although they remain a possibility. Thus, the usefulness of partial selections in multilevel models lies in the possibiliity of avoiding infeasible problem formulations and in improving objective function values relative to a binary variable formulation of the same system.

## The Problem

The use of partial selections is of interest because the costs of generating new alternatives are avoided for situations where none of the initial alternatives is

satisfactory to the decision maker.[1] While partial selections reduce analytical costs, infeasible or suboptimal alternatives may be generated. The question becomes whether accepting the possible error in cost estimates for partial selection alternatives is preferable to incurring the expense of resources required by the evaluation process.

This will depend on the nature of the evaluation process as well as the magnitude and direction of error of the partial selection alternative cost estimates. If the partial selection alternatives are in fact infeasible, plans generated using the technique will not be operable. Suboptimal partial selection alternatives are at least feasible and thus have output combinations that are in fact attainable at the predicted cost. In a sense, when a partial selection alternative is generated by a multilevel model, it is preferable to the initial optimal alternatives that are not selected. A decision maker may also prefer a suboptimal partial selection alternative over the choices of optimal alternatives. Thus, suboptimality of partial selection alternatives may be less critical than infeasibility in determining the utility of the technique.

The problem becomes how to determine the degree of infeasibility or suboptimality of partial selection alternatives without explicitly evaluating them.

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[1]The decision maker, or selector of alternatives, could be a multilevel computer model.

## Objectives

The objectives of this study were to:

1. Develop hypotheses based on economic theory regarding the feasibility and suboptimality of partial selection alternatives;
2. Develop a procedure to test the hypotheses empirically; and
3. Test the hypotheses for a representative case situation.

# CHAPTER TWO

## THEORETICAL IMPLICATIONS OF PARTIAL SELECTIONS

The purpose of this chapter is to develop hypotheses concerning the factors influencing the magnitude and direction of error in cost estimates of partial selection alternatives. The hypotheses will then be tested for a case forest planning situation.

The discussion will initially focus on a time horizon constrained model of the forest production system. The outputs of alternatives are taken as the total output quantities over the planning horizon without regard for the distribution of production over time. In actuality, timing of output flows may require special consideration and will be examined later.

Although a partial selection may involve any number of alternatives, only combinations of two alternatives will be discussed. This should present no problem, as any partial selection can be thought of as a change in production from an initial optimal alternative. The convention here will be to examine changes from the lower budget alternative of the pair of initial alternatives being combined.

## Feasibility of Partial Selection Alternatives

An intuitive example will be used to demonstrate that partial selections of forest alternatives generate output combinations that can in fact be produced at the predicted cost (partial selection alternatives are at least feasible).

Consider the 10-90 partial selection of the two alternatives shown in Table 1. For simplicity the forest is assumed to be divided into two analysis areas. The acreage in each analysis area must be assigned to one of three prescriptions. Partial selection outputs, benefits and cost are calculated as a weighted average of the original values. A land allocation for the partial selection alternative can be generated in the same manner from the land allocations corresponding to the original alternatives.

The proportionality conditions of linear programming (Budnick, Mojena and Vollmann 1977) guarantee that output quantities and cost will change by the same proportion as the acreage assigned to a given prescription. Thus the land allocation for the partial selection alternative will produce the required output levels at the cost derived by fractionally combining the budget requirements of the original alternatives. In other words, the partial selection alternative is feasible.

It can also be shown that, for the example, the partial selection alternative will be suboptimal. Each of the

Table 1. Outputs, Benefits, Cost and Land Allocation for a 10-90 Partial Selection of Two Forest Alternatives.

Outputs, Benefits and Cost

| | Alternative 1 | Alternative 2 | 10-90 Partial Selection |
|---|---|---|---|
| Fraction chosen | .1 | .9 | .1(120) + .9(300) = 282 |
| Sawtimber (MMBF) | 120 | 300 | .1(4743) + .9(9840) = 9330 |
| PNW (M$) | 4743 | 9840 | .1(4743) + .9(9840) = 9330 |
| Cost (M$) | 1158 | 4039 | .1(1158) + .9(4039) = 3751 |

Land Allocation

Acreage Assigned by Prescription (M Acres)

| | Alternative 1 | | | Alternative 2 | | | 10-90 Partial Selection | | |
|---|---|---|---|---|---|---|---|---|---|
| Analysis Area | Prescription A | B | C | Prescription A | B | C | Prescription A | B | C |
| 1 | 100 | | | | 50 | 50 | .1(100) =10 | .9(50) =45 | .9(50) =45 |
| 2 | | 400 | 100 | 500 | | | .9(500) =450 | .1(400) =40 | .1(100) =10 |

original alternatives assigns the acreage in the forest to three prescriptions. The partial selection alternative allocates the acreage to the six available prescriptions. Since each prescription assigned to acreage would constitute a basis variable in a linear programming solution corresponding to the given land allocation, the number of variables in the basis increases by three for the partial selection alternative. Optimal LP solutions must have the same number of variables in the basis as the number of rows (constraints) included in the model (Budnick, Mojena and Vollmann 1977). Since utilization of constraint capacity also conforms to proportionality conditions, no new slack or surplus variables enter or leave the solution corresponding to the partial selection alternative. Since the number of variables in the basis exceeds the number of rows for the partial selection alternative, it must be suboptimal. In other words, by reassigning acreage to prescriptions, the partial selection output quantities could be produced at a lower cost than predicted.

If the original alternatives assign acreage to identical prescriptions but in different quantities, the number of variables in the basis will remain unchanged for any partial selection and the alternative will be optimal. For this special case, output quantities and cost change equiproportionately in moving from a proposed alternative to a partial selection output combination due to the

proportionality conditions of LP. The conditions are no longer guaranteed when different prescriptions are included in the alternatives being combined, as in Table 1.

Feasibility of partial selection alternatives can also be demonstrated graphically. Figure 3 illustrates the special case of partial selection of alternatives with the same budget requirement.

Any partial selection output combination will lie on the line segment between points 1 and 2 representing the two initial alternatives. The corresponding predicted cost is derived as:

$$B_P = sB_1 + (1 - s)B_1$$

where $B_1$ is the cost of producing either initial output combination. For a strictly convex production possibilities curve, any partial selection generates a corresponding output combination that lies inside the curve for its associated predicted cost. Thus the partial selection alternative must be feasible but suboptimal. For points such as T, closer to an optimal alternative, the degree of suboptimality is expected to be less due to the convexity of the production possibilities curve.

Optimal partial selection alternatives would be generated for this special case only for a linear production possibilities curve over the range of output change. This corresponds to equiproportional changes in output quantities

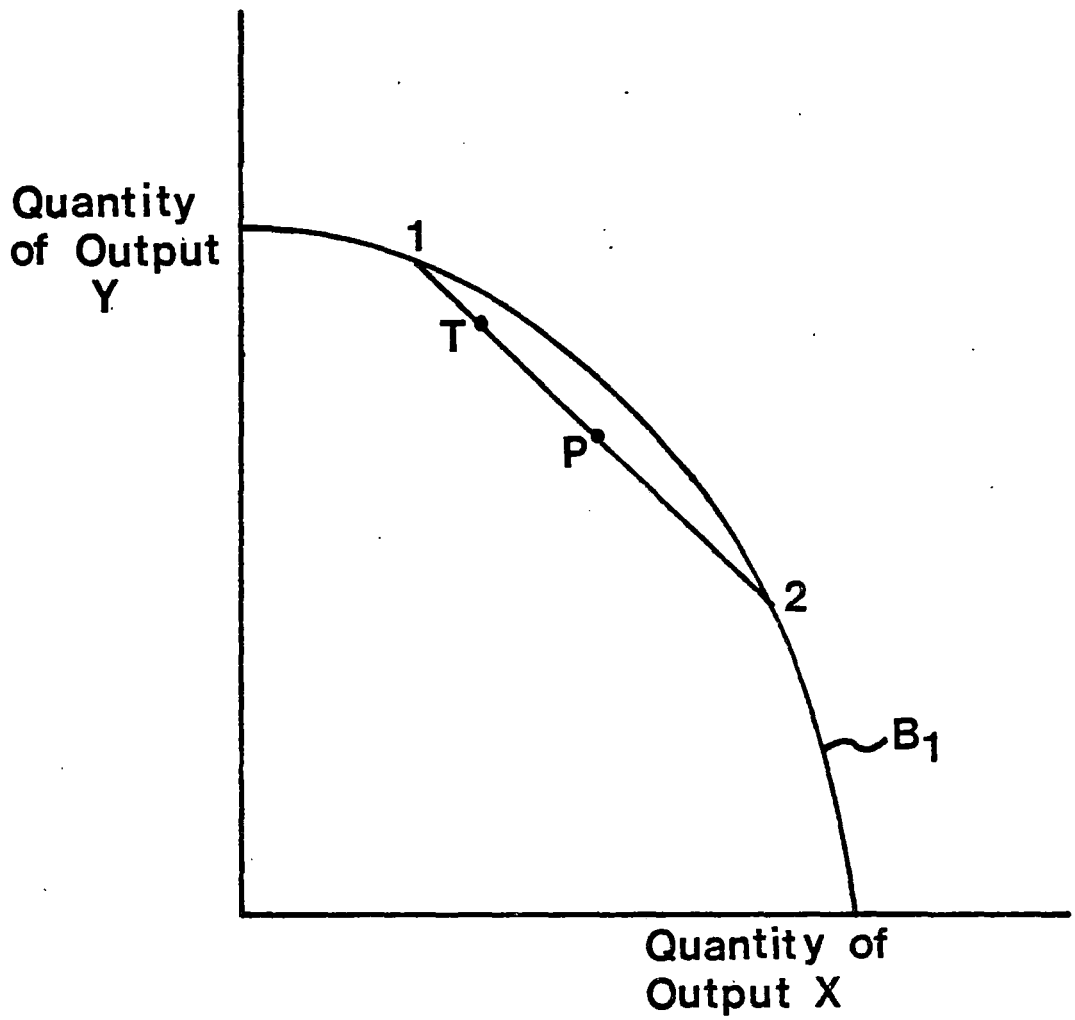


Figure 3. Partial Selection of Two Alternatives with the Same Budget Requirements.

and cost for the case of alternatives with identical prescriptions described previously.

## Returns to Outlay and Marginal Cost

Equiproportional changes in output quantities and cost required for optimality of partial selection alternatives translates to constant returns to a single factor, the required budget. Here the required budget is used as an aggregate measure of inputs.[2]

Changes in output related to changes in cost (budget) have been referred to as returns to outlay (Russell and Wilkinson 1979). Since in forest planning the decision maker will probably select desired output levels and predict the required budget, it is more useful to relate changes in cost to the change in output quantities required by a partial selection. This leads to the use of the dual formulation of returns to outlay, marginal cost.

The marginal cost function for a given product can be derived by considering the change in returns to outlay as the output quantity is increased. Figure 4 shows total and marginal cost functions for three returns to outlay cases. The first case illustrates that for increasing returns to outlay, the corresponding marginal cost function must be decreasing. For the second case, constant returns to outlay

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[2]An additional input not captured by budget, land, is assumed to be fixed in quantity for a given forest and cannot be treated as a decision variable.

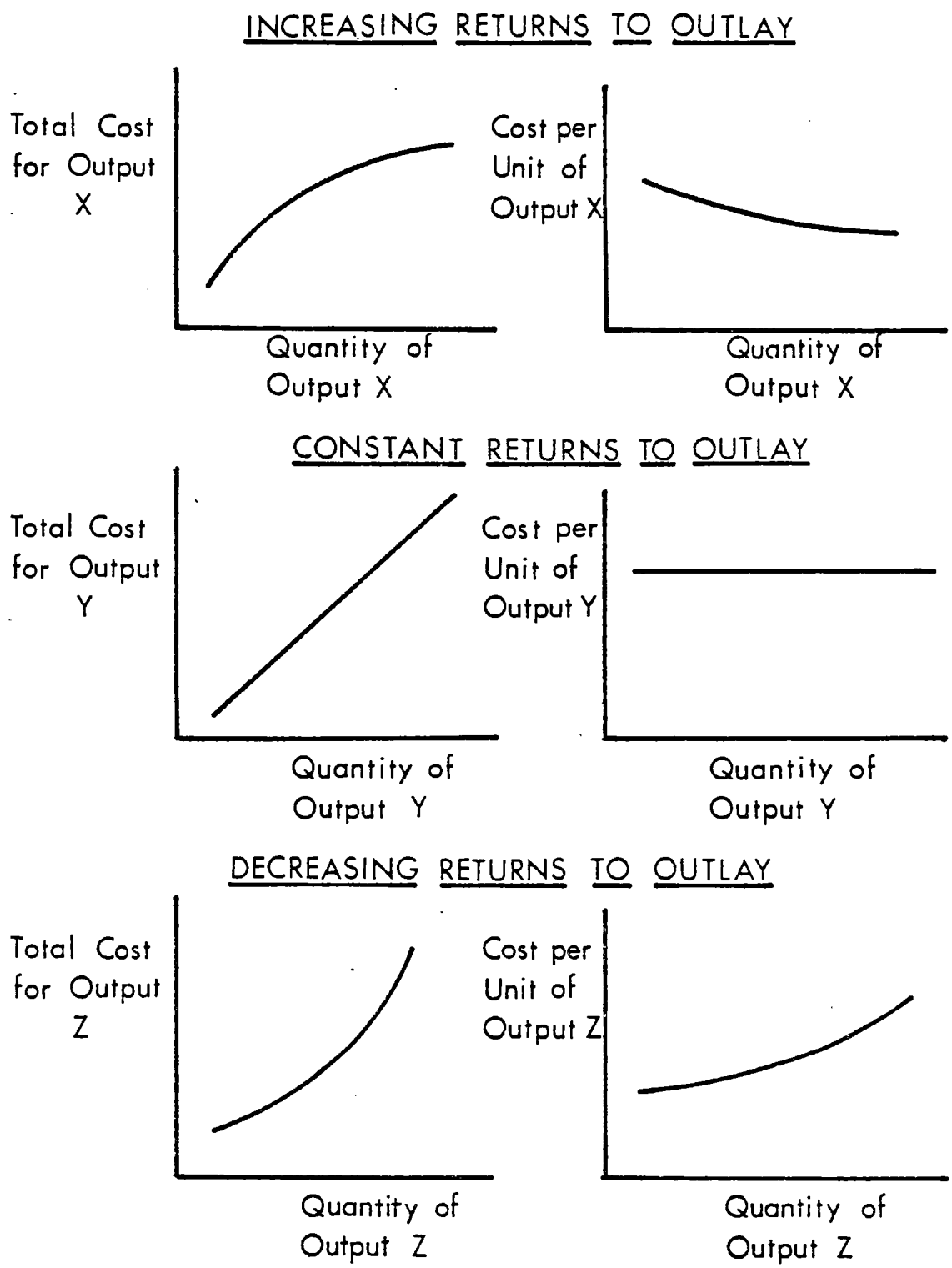
INCREASING RETURNS TO OUTLAY
Total Cost for Output X
Quantity of Output X
Cost per Unit of Output X
Quantity of Output X
CONSTANT RETURNS TO OUTLAY
Total Cost for Output Y
Quantity of Output Y
Cost per Unit of Output Y
Quantity of Output Y
DECREASING RETURNS TO OUTLAY
Total Cost for Output Z
Quantity of Output Z
Cost per Unit of Output Z
Quantity of Output Z

Figure 4. Total and Marginal Cost Functions for Three Returns to Outlay Cases.

are associated with a linear total cost function and marginal cost must be constant.

For the third case, it is demonstrated that decreasing returns to outlay generates a marginal cost function that must be increasing. In the discussion that follows, increasing marginal cost functions will be assumed. This assumption seems most reasonable because an LP algorithm brings into solution successively higher cost prescriptions in determining the land allocation that produces a given output combination for a cost minimization objective. In addition, it also seems intuitively reasonable that partial selection of forest alternatives will generally involve changes in outputs over production ranges where returns to outlay are decreasing.

## Suboptimality of Partial Selection Alternatives

It has been shown that partial selections of forest alternatives are at least feasible. The discussion will now center on factors influencing the error, or degree of suboptimality of partial selection alternatives, for two different models of the forest production system. The first model assumes independence of marginal cost functions for forest outputs. A production system with interrelated marginal cost functions will then be examined.

Independent Production of Forest Outputs

It is important to note here that the independence being assumed relates to marginal cost functions of the forest outputs. Thus the marginal cost function for a given output is not influenced by level of production of the other outputs. While partial selection output levels are in fact linked through s, the partial selection proportion, the cost of changing a given output is independent of other production levels.

It has been shown previously that partial selection alternatives are optimal only when returns to outlay are constant (marginal cost is constant). This follows from implicitly costing production changes from an initial optimal output combination at a constant proportion of the total cost change between the original alternatives. Cost of a partial selection alternative is derived as:

$$\text{PREDICTED COST} = sB_1 + (1 - s)B_2 \qquad (1)$$

for combining the initial alternatives 1 and 2 with budget requirements $B_1$ and $B_2$.

The budget requirement for this second alternative, $B_2$, is equal to $B_1$ plus the cost of the production change necessary to generate the output combination of the second alternative. Substituting to eliminate $B_2$:

$$\text{PREDICTED COST} = sB_1 + (1-s)[B_1 + MC_X(X)dx + MC_Y(Y)dy]$$

where X and Y represent the two output variables and the marginal cost function for a given product is dependent only on its own output level (assuming independent production and constant factor prices). Using $TC_i$ to represent the total cost of the production change between alternatives 1 and 2 for output i:

$$\text{PREDICTED COST} = B_1 + (1-s)TC_X + (1-s)TC_Y \qquad (2)$$

where cost is predicted as the first budget plus a proportion of the total cost of the production change required to move from the lower cost output combination $(X_1,Y_1)$ to $(X_2,Y_2)$.

Now the following identities will be used to eliminate s:

$$s = \frac{X_2 - X_P}{X_2 - X_1}(X_P - X_1) \qquad 1-s = \frac{X_P - X_1}{Y_2 - Y_1}(Y_P - Y_1)$$

where $X_P$ is the partial selection output level for product X. It can be seen that as $X_P$ approaches $X_2$, s goes to zero and all of the second alternative is chosen. A similar pair of expressions exists for output Y.

Substituting for (1-s) in equation (2):

$$\text{PREDICTED COST} = B_1 + \frac{TC_X}{X_2 - X_1}(X_P - X_1)$$

$$+ \frac{TC_Y}{Y_2 - Y_2}(Y_P - Y_1)$$

The total cost term, when divided by the difference in output levels between the proposed alternatives, yields a constant that is used as the predictor of the cost of output changes required by a partial selection alternative. This constant for a given product is the average marginal cost. Substituting $AMC_i$ for the average marginal cost of output i, the desired result is obtained:

$$\text{PREDICTED COST} = B_1 + AMC_X(X_P - X_1) + AMC_Y(Y_P - Y_1) \quad (3)$$

Here output changes from the lower cost alternative are assumed to incur a constant per unit cost in absence of knowledge of the true form of the marginal cost functions.

If marginal cost functions could be derived, actual cost of the partial selection output combination could be determined by the following equation:

$$\text{ACTUAL COST} = B_1 + \int_{X_1}^{X_P} MC_X(X)dx + \int_{Y_1}^{Y_P} MC_Y(Y)dy \quad (4)$$

where form of the marginal cost functions is critical in determining the cost of output changes. The relationship between form of marginal cost functions and the error in cost estimates (degree of suboptimality) of partial selection alternatives will be described for several nondecreasing types of functions.

## Form of Marginal Cost Functions and Partial Selection Suboptimality

As an introductory example consider the two alternatives shown in figure 5. Any partial selection output combination will lie along the line segment between points 1 and 2 and require a production increase for both outputs relative to the low budget alternative 1. The error, or degree of suboptimality of a partial selection alternative, is:

$$\text{ERROR} = \text{PREDICTED COST} - \text{ACTUAL COST} \tag{5}$$

Substituting equations (3) and (4) into equation (5):

$$\text{ERROR} = \left[AMC_X(X_P - X_1) - \int_{X_1}^{X_P} MC_X(X)\,dx\right] + \left[AMC_Y(Y_P - Y_1) - \int_{Y_1}^{Y_P} MC_Y(Y)\,dy\right] \tag{6}$$

Figure 6 shows the graphical derivation of the total error for a partial selection of the two alternatives shown in

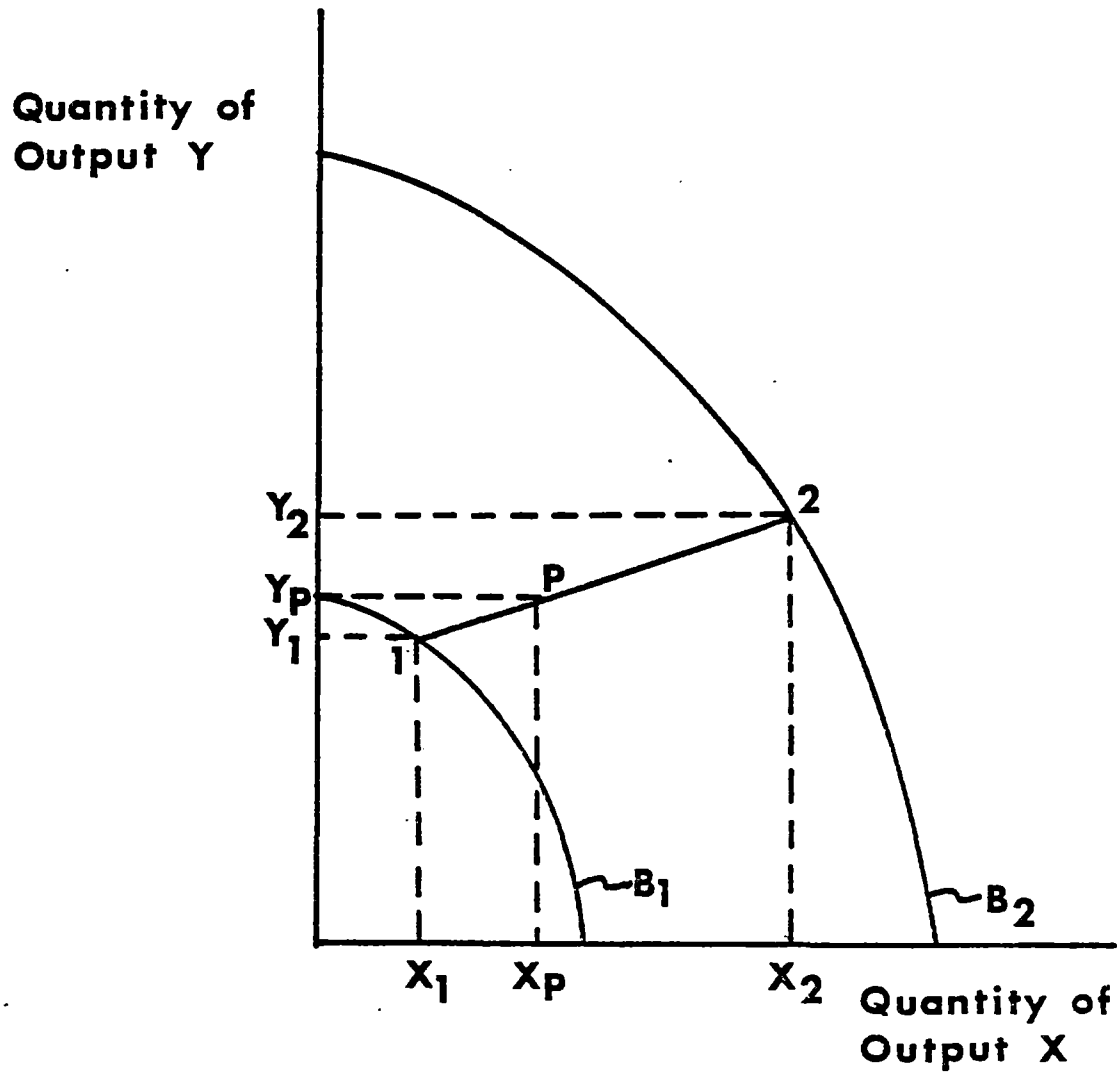


Figure 5. Partial Selection Alternative Requiring and Increase in Two Outputs Relative to the Lower Budget Alternatives.

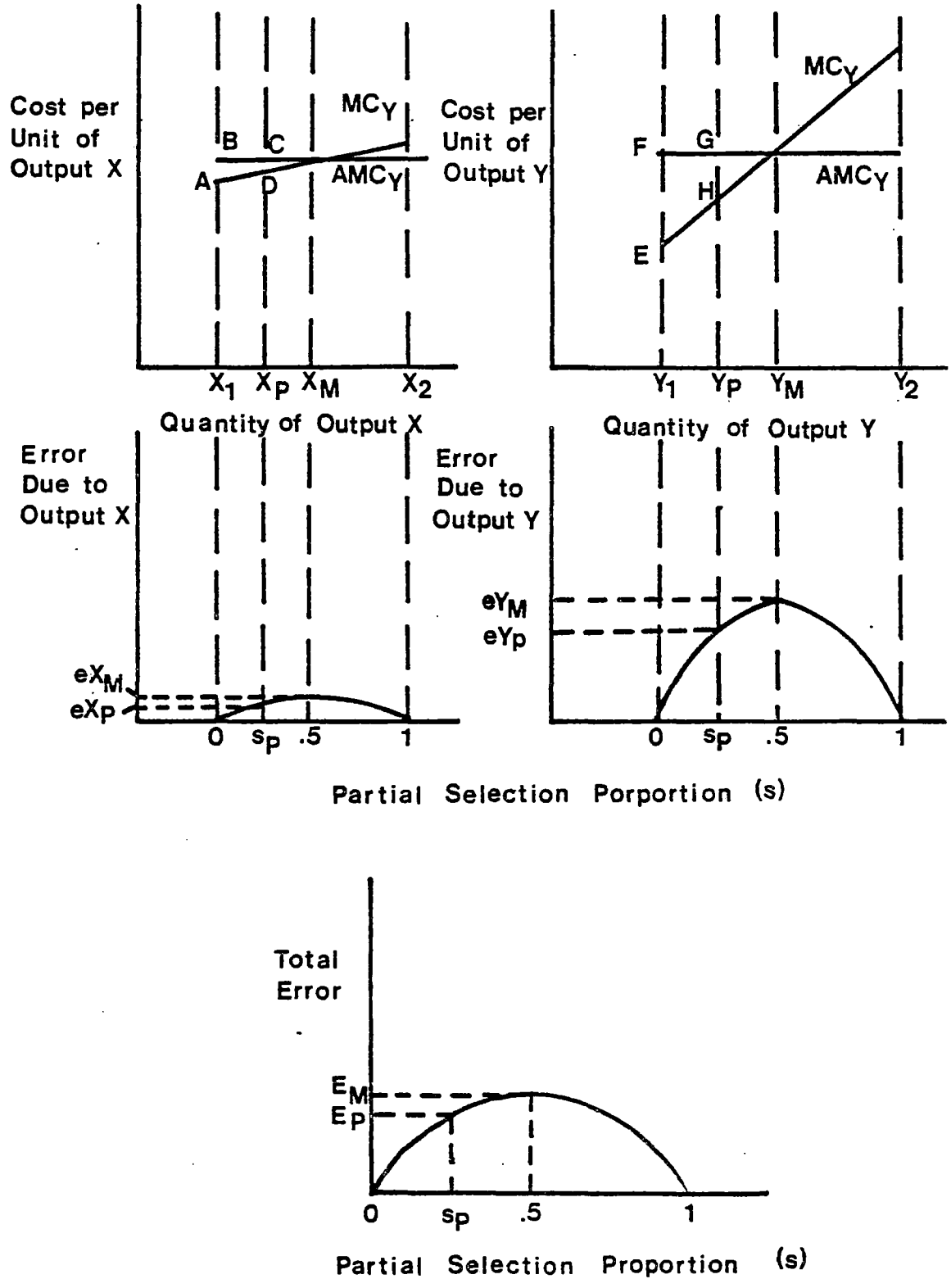


Figure 6. Determination of Partial Selection Error for Two Outputs with Linear Marginal Cost Functions.

figure 5. Initially linear marginal cost functions will be assumed for simplicity. In the figure, average marginal cost for each product is a horizontal line, and the area under the average marginal cost line and the marginal cost function are the same for a given product between the output quantities of the two original alternatives, $X_1$ to $X_2$ and $Y_1$ to $Y_2$.

For the given pair of marginal cost functions, and any increasing marginal cost functions, $AMC_i$ is greater than $MC_i$ for initial production increases from the alternative 1 combination, $(X_1,Y_1)$. Thus, the error for a given product is positive (toward suboptimality) and accumulates as the required output level increases up to a maximum at the point where MC intersects AMC from below.[3] For linear marginal cost functions, this occurs at the production combination required for equal selection of the two alternatives (s=.5). Increases in production beyond this point involve cancellation of error such that at the output levels of alternative 2, error is zero (it is an optimal alternative).

For the partial selection output combination $(X_p,Y_p)$ in the figure, error from output X is the area ABCD,

---

[3]The intuitive proof of feasibility shown previously in Table 1 holds because linearly combining the land allocations, outputs and cost generates constant marginal cost functions. The suboptimality that followed based on the number of basis variables corresponds here to increasing marginal cost functions.

corresponding to $eX_p$ on the vertical axis of the graph of the error due to output X. Similarly the partial selection error from output Y is derived as the area EFGH, or $eY_p$ on the vertical axis of the graph of error due to output Y.

Because the marginal cost functions are assumed to be independent, the two error functions can be summed to generate a total error function. For any given value of s, error is read as the height of the function.

For increasing marginal cost functions, the error function for a given product must be zero for the two original alternatives and reach a single maximum somewhere in the interior region of possible output levels. For this example, two additional results, that cannot be generalized to nonlinear marginal cost functions, are that the error functions are symmetrical and the maximum error occurs for equal (50-50) selection of the original alternatives.

Figure 7 illustrates the derivation of the total error function for two products with different forms of nonlinear marginal cost functions. The function for output X has a nearly constant marginal cost up to the point of maximum error at output level $X_M$. Error cancels relatively quickly past this point due to the increasing slope of the marginal cost function.

Conversely, the marginal cost function for output Y generates a relatively large error for initial production changes from $Y_1$. Here the maximum error occurs at $Y_N$,

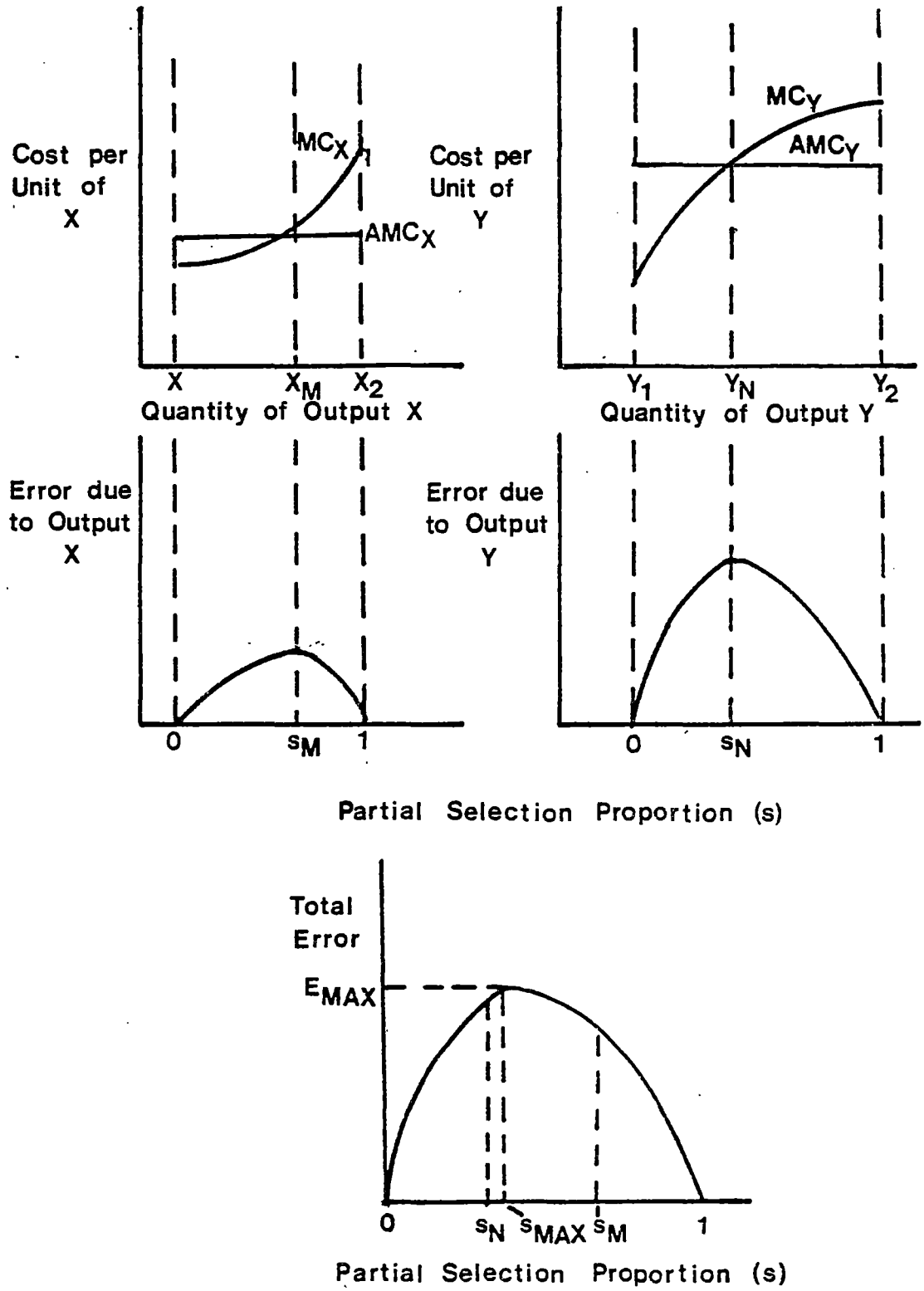


Figure 7. Determination of Partial Selection Error for Two Outputs with Nonlinear Marginal Cost Functions.

closer to $Y_1$ than to $Y_2$. Error cancels slowly past the output level $Y_N$ due to the decreasing slope of the marginal cost function. For this example, output Y contributes more to the total error function than does output X. Thus the total error function has a maximum, $E_{MAX}$, for the partial selection proportion $s_{MAX}$ that is much closer to the value $s_N$ that maximizes error for output Y than to $s_M$, the proportion that maximizes error for output X.

Before continuing it should be noted that the possible forms of marginal cost functions that could be encountered are endless. Nonetheless, the general concepts presented here should be helpful guidelines in dealing with various functional forms. The marginal cost functions for outputs X and Y in these two examples were selected to provide a best and worst case type of scenario in terms of partial selection error, against which other linear or nonlinear functional forms may be compared.

It is not always the case that all output quantities increase in moving toward the higher budget alternative.[4] Figure 8 illustrates a situation where output X will decrease in moving from the lower budget alternative 1 toward alternative 2. For any partial selection of the two alternatives, the predicted cost is of the form derived earlier:

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[4]For initial alternatives with equal budget requirements, at least one output quantity must decrease in moving from one optimal alternative toward the other.

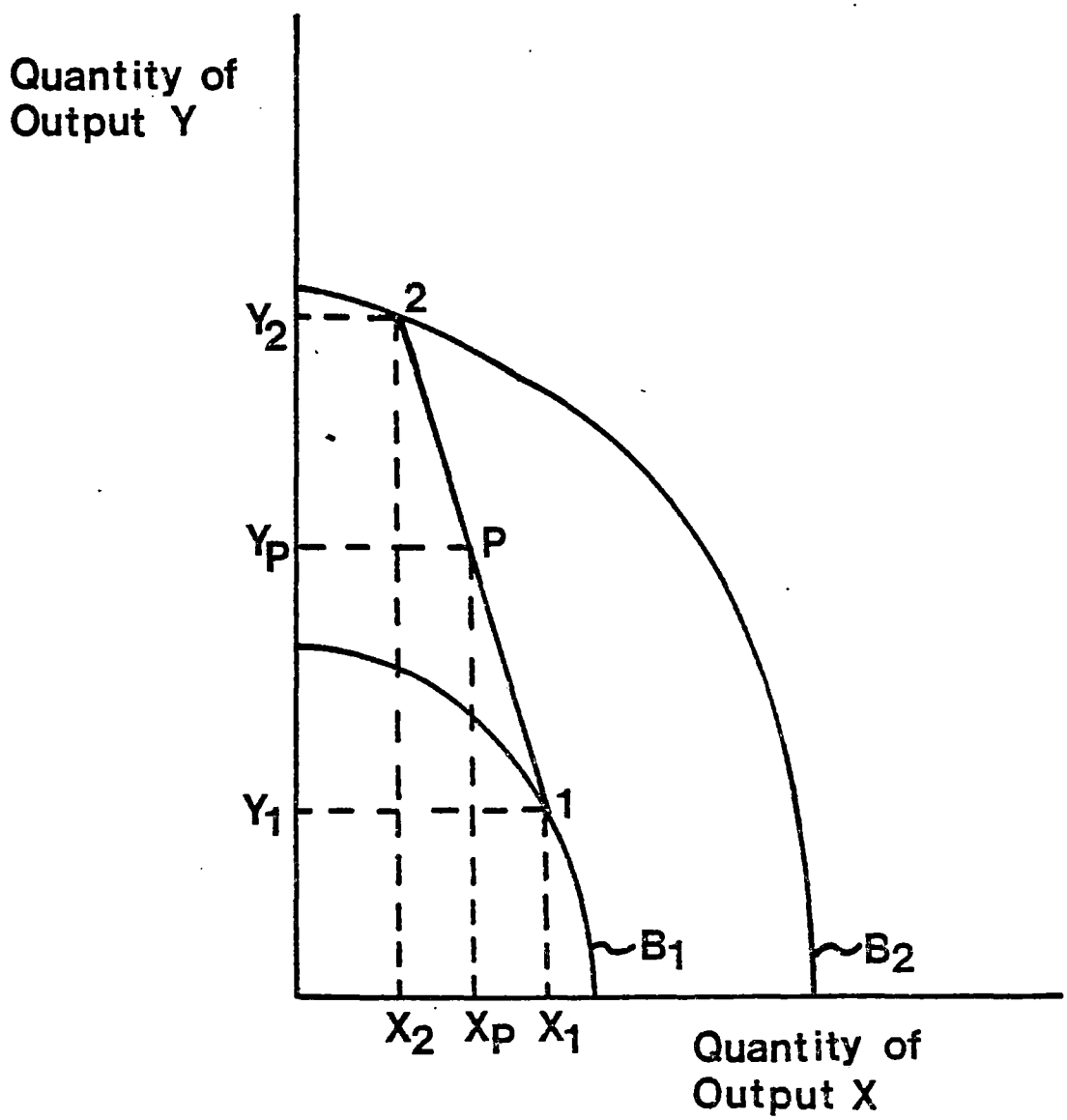


Figure 8. Partial Selection Alternative That Requires a Decrease in One Output.

$$\text{PREDICTED COST} = B_1 - AMC_X(X_1 - X_P) + AMC_Y(Y_P - Y_1) \quad (7)$$

where now a cost savings is obtained for the decreased production of output X.

Now, for the actual cost,

$$\text{ACTUAL COST} = B_1 - \int_{X_P}^{X_1} MC_X(X)\,dx + \int_{Y_1}^{Y_P} MC_Y(Y)\,dy \quad (8)$$

where the savings from decreased production of output X is properly costed according to marginal cost.

Subtracting equation (8) from equation (7) to obtain an error function:

$$\text{ERROR} = \int_{X_P}^{X_1} MC_X(X)\,dx - AMC_X(X_1 - X_P) + AMC_Y(Y_P - Y_1) - \int_{Y_1}^{Y_P} MC_Y(Y)\,dy \quad (9)$$

which is not the same as equation (6) derived previously.

Output Y is treated in the same manner as before for production increases. Examining the error due to output X:

$$\text{ERROR}_X = \int_{X_P}^{X_1} MC_X(X)\,dx - AMC_X(X_1 - X_P)$$

For increasing marginal cost functions this error for output X must be positive. Now the cost savings due to a production decrease is being underestimated, and once again error is in the direction of suboptimality. Thus, error for individual products is always additive regardless of the direction of output changes. Hence, including additional costed outputs in an LP model representing the forest production system can only increase the degree of suboptimality of any partial selection alternative generated using the model.

The hypotheses developed under the assumption of independent marginal cost functions are:

1. Partial selection alternatives are feasible for products with nondecreasing marginal cost functions.
2. For increasing marginal cost functions, partial selection alternatives will be suboptimal. This follows from implicitly assuming a constant marginal cost, i.e., average marginal cost, for output changes between optimal alternatives.
3. Error for an output or a partial selection has a zero value for the output quantities of the original alternatives and has a single maximum in the interior region of possible production change.

4. The magnitude of error generated by a given output will decrease as the marginal cost function approximates a constant marginal cost over the range of production change being considered.
5. For independent marginal cost functions, total error for a multiproduct system is a summation of the individual product errors, with the relative contribution of each product being determined by item 4 above. Error is additive for products with output levels being increased or decreased if all marginal cost functions are increasing in the region of production change.

Joint Production of Forest Outputs

We now consider a more realistic model of the forest production system involving jointness in production of outputs. Jointness arises due to two factors: commonality of inputs and output interactions. For example, fertilization to increase timber yields will also raise forage production. The input is jointly used by the two products, i.e., both production functions contain a term for the fertilizer input. Not having to purchase and apply the input twice results in a cost savings, i.e., the marginal cost functions shift downward due to the common input usage.

An upward shift due to commonality is also a possibility, although the examples seem to be far less numerous. Using herbicides to decrease the cost of timber operations

could also decrease forage production. Here the production functions both contain terms for the herbicide treatment, although it can be thought of as an input only for timber production. It could be considered to be a negative externality in the production of forage. Thus the marginal cost function for forage is shifted upward due to herbicide usage.

Output interactions arise from ecological interactions between products. In other words, the production function for a given output contains terms for other output variables. These interactions can be in either direction in terms of shifting marginal cost functions. For example, increasing the recreation use on a forest may have negative impacts on wildlife populations or timber production. Forage production can have a positive influence on wildlife populations. The various interactions between wildlife populations are also examples of product interactions.

As a result of commonality of inputs and product interactions, the cost of a treatment cannot be allocated to a single output (Silberberg 1978). For example, a prescribed burn that simultaneously increases several products has a cost that cannot be allocated to the outputs it has an impact on. Additional examples include law enforcement, fire protection, and maintenance of soil and water quality. Examples of treatments that affect the production of a single output exclusively seem scarce.

For a joint production system, we begin by describing a minimum cost function $C^*$ for the production of n outputs using m inputs (Silberberg 1978):

$$C^*(r_1, r_2, \ldots, r_m, Y_1, Y_2, \ldots, Y_n)$$

where $r_1$ through $r_m$ are factor prices for m inputs, and the $Y_i$'s represent output quantities for n products. The difference between this and the independent system discussed previously now arises as the marginal cost function for a given product will be a function of other output quantities as well as its own output level. Taking first partials and holding factor prices constant,

$$\frac{C^*}{Y_i} = MC_i(Y_1, \ldots, Y_n), \qquad i = 1, \ldots, n$$

where $MC_i$ is the marginal cost function for output i. For this joint production system, the marginal cost function for a given product may contain the output variables for any or all of the other products.

If the marginal cost function for output i is dependent on the quantity being produced of output j, then the second cross partial derivative with respect to $Y_j$ will be nonzero:

$$\frac{{}^2C^*}{Y_j Y_i} = \frac{MC_i}{Y_j} \neq 0$$

In other words, the marginal cost of output i is influenced by the quantity being produced of output j.

This situation is shown in figure 9. Two products X and Y are being jointly produced. Initially, assume output Y is being produced at the quantity $\underline{Y}$, generating the marginal cost function $MC_X(X,\underline{Y})$. At greater output levels of Y, for example Y in the figure, two possible situations are shown. If there is a competitive relationship between the products, the higher marginal cost function $MC_X^1(X,Y)$ is appropriate. The lower function $MC_X^2(X,Y)$ is for a complementary relationship between the outputs.

It was shown by Hof (in process) that the cost of a production change can be represented by a line integral of the marginal cost functions, and that while total cost is not dependent on the path of integration, the allocation of cost to individual products is path dependent. Thus there is an infinite number of possible integration paths, or cost allocations, for a given production change.

Two possible expressions for the actual cost of a partial selection output combination $(X_p,Y_p)$ are derived as:

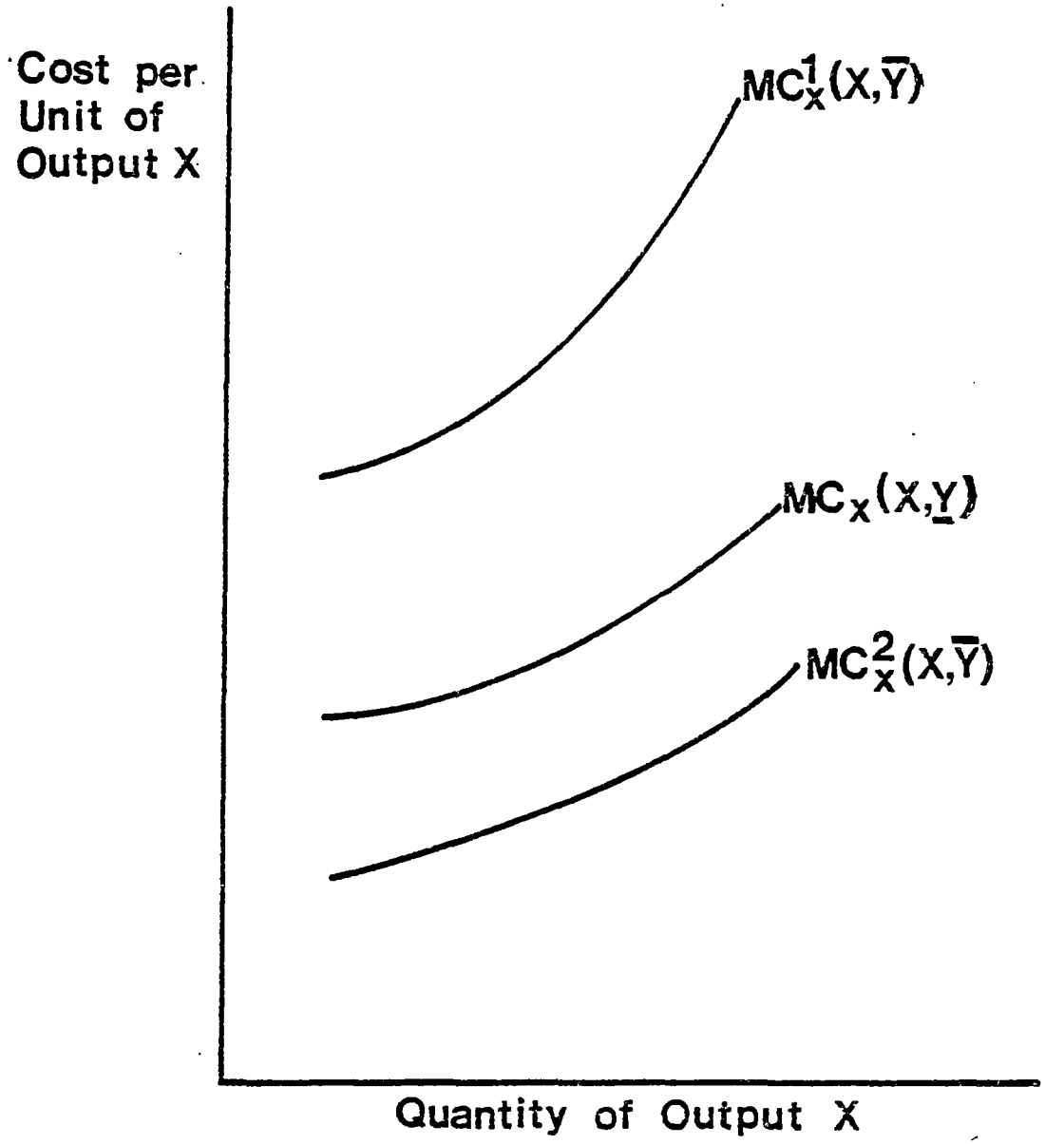


Figure 9. Dependent Marginal Cost Functions for Two Outputs in a Joint Production System.

$$\text{ACTUAL COST} = B_1 + \int_{X_1}^{X_P} MC_X(X,Y_1)\,dx + \int_{Y_1}^{Y_P} MC_Y(X_P,Y)\,dy \qquad (10a)$$

$$\text{ACTUAL COST} = B_1 = \int_{X_1}^{X_P} MC_X(X,Y_P)\,dx + \int_{Y_1}^{Y_P} MC_Y(X_1,Y)\,dy \qquad (10b)$$

where the combination $(X_P,Y_P)$ constitutes a production increase relative to the low budget combination $(X_1,Y_1)$. The first equation involves integrating the marginal cost function for output X first, with the quantity of output Y held constant at $Y_1$, and then integrating the marginal cost for output Y, with the quantity of output X held constant at $X_P$. The second integrates the function for Y first, holding output X at $X_1$, and then the function for X, holding Y constant at $Y_P$. These are two of the possible paths of integration.

An equivalent pair of expressions can also be derived for the predicted cost of the partial selection output combination:

$$\text{PREDICTED COST} = sB_1 + (1 - s)B_2$$

Substituting for the joint cost of the second alternative,

$$\text{PREDICTED COST} = sB_1 + (1 - s)\left[B_1 + \int_{X_1}^{X_P} MC_X(X,Y_1)\,dx + \int_{Y_1}^{Y_P} MC_Y(X_P,Y)\,dy\right]$$

and collecting terms,

$$\text{PREDICTED COST} = B_1 + (1 - s)\left[\int_{X_1}^{X_P} MC_X(X,Y_1)\,dx + \int_{Y_1}^{Y_P} MC_Y(X_P,Y)\,dy\right] \qquad (11a)$$

In equation (11a), the marginal cost function for output X is integrated first. Alternatively, integrating the marginal cost function for output Y first, a second expression is generated:

$$\text{PREDICTED COST} = B_1 + (1 - s)\left[\int_{X_1}^{X_P} MC_X(X,Y_P)\,dx + \int_{Y_1}^{Y_P} MC_Y(X_1,Y)\,dy\right] \qquad (11b)$$

Once again marginal costs of production changes are being incorrectly assumed to be constant, as in the case of independent marginal cost functions. Now, however, cost cannot be allocated to individual products. As a result of

the predicted and actual costs being non-allocable, the partial selection error also cannot be allocated to products.

This leads to the determination of error illustrated in figure 10. For the two joint products, X and Y, output levels of a partial selection will be linked through the choice of s, the partial selection proportion. Thus there will be some marginal cost function associated with increasing the value of s, used here as an aggregate measure of production changes from the lower budget alternative. Cost is predicted to be a constant proportion of the joint cost of changing production from $(X_1,Y_1)$ to $(X_2,Y_2)$. Thus error is generated in the same manner as before, only now output changes are expressed through a single variable, s.

The degree of suboptimality of the partial selection alternative is now dependent on the form of the marginal cost function for s. This functional form is in turn dependent on the forms of the individual output marginal cost functions (without production of the other output) as well as the interactions between them.

As stated, error cannot be attributed to individual products. In addition, error does not necessarily increase with the number of costed outputs in a multiproduct system. This is because interactions of product marginal cost

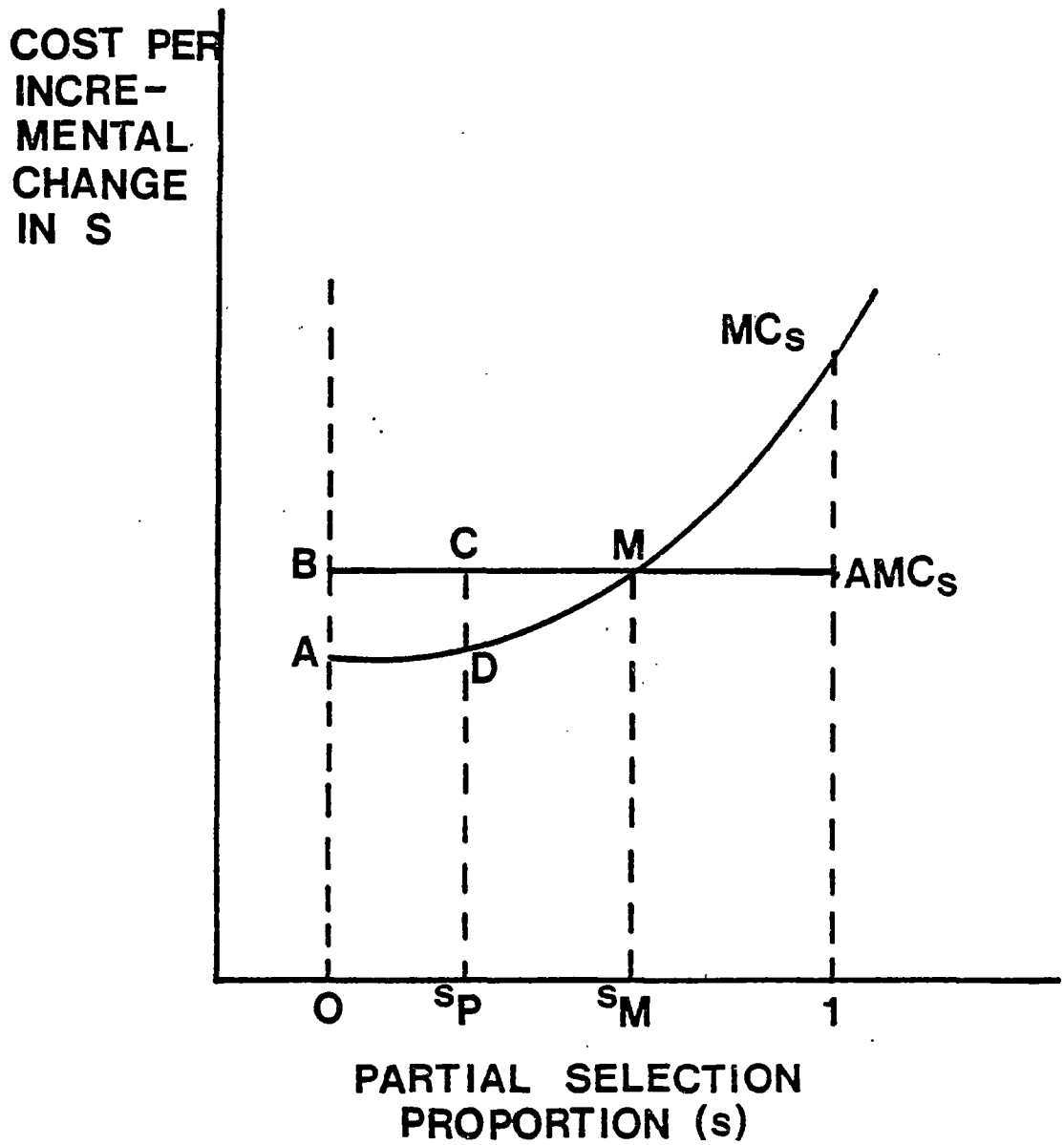


Figure 10. Determination of Partial Selection Error for Two Joint Products.

functions that influence the aggregate marginal cost function cannot be predicted.[5]

### Dynamic Considerations for Partial Selections

As a final point in the discussion of the theoretical implications, a dynamically constrained model of forest production will be considered. In this case the temporal distribution of outputs in the original alternatives is critical, as the partial selection output quantities now must be set for each period in the planning horizon.

In the previous model, which was constrained only by the planning horizon, partial selection output requirements translated into a single constraint for required total production of each output in a cost minimization LP formulation. In the dynamically constrained model, a single constraint is required for each output and time period combination. Since the individual time period constraints for a product, when considered in sum, also constrain total production over the planning horizon, the time period requirements result in an increase in the number of constraints as compared to the model constrained only by the planning horizon. Since adding constraints can only decrease the feasible region of an LP problem formulation, the

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[5]Predicting the interactions of marginal cost functions would have to be done in terms of impact on functional form, and not just on whether the functions are shifted upward or downward.

value of the objective function can only be unchanged or decreased in terms of optimality as compared to the less restrictive time horizon constrained model. Where the objective is to minimize the cost of producing the output quantities of a partial selection alternative, the actual cost for the dynamically constrained model will be at least equal to the actual cost determined in a time horizon constrained model.

Cost is predicted in the same manner as a fractional combination of the original alternative budget requirements and must be the same for the two models being discussed for a given pair of optimal alternatives being combined. Thus, since predicted cost is the same and actual cost can be increased by adding the time period constraints, degree of suboptimality of a partial selection alternative may be decreased in a dynamically constrained model relative to a time horizon constrained model.

The same logic as above applies to other intertemporal constraints such as nondeclining even flow, sustained yield constraints, or ending inventory constraints. Actual cost can be increased due to these constraints in a cost minimization formulation to produce partial selection output quantities. If actual cost is increased by intertemporal constraints, partial selection error must decrease since predicted cost is not influenced by the constraint conditions.

The hypotheses developed earlier as modified to reflect considerations of joint production and dynamics are:

1. Partial selection alternatives are feasible for aggregate marginal cost functions that are increasing. This result is guaranteed for forest planning using LP. Interactions of marginal cost functions make prediction of feasibility based on product marginal cost functions impossible. Although feasibility is guaranteed for an increasing aggregate marginal cost function, increasing functions also lead to suboptimal partial selection alternatives. This follows from implicitly costing output changes (changes in s) at a constant (average marginal cost).
2. For an aggregate or product marginal cost function that increases with s, the error function for a partial selection will have a value of zero for the original alternative output quantities (s = 0,1) and have a single maximum in the interior region of possible production change.
3. Error functions are symmetrical for linear marginal cost functions, but not for other possible functional forms.
4. The magnitude of error for a partial seelction will decrease as the marginal cost, expressed as a

function of s, approximates a constant function over the range of production change being considered.

5. Error tends to be additive between products with increasing marginal cost functions regardless of the direction of output changes. For independent marginal cost functions, the error for a partial selection is simply the summation of individual product errors. Interactive marginal cost functions can negate additivity of product errors, i.e., adding a new costed output to a model formulation does not necessarily lead to a larger magnitude of error for a given partial selection, because the form of marginal cost functions for the original products may be altered.
6. If production is joint, error cannot be allocated to individual products.
7. Constraints on intertemporal output flows may decrease the error of partial selection alternatives, but cannot increase error.

The results of the empirical analysis to follow will be examined and applied to the validity of the developed hypotheses. While the general analysis to be described is designed to address as many of the postulates as possible for the resources available, the validation of the entire set is beyond the scope of this study.

# CHAPTER THREE

# PROCEDURE

The basic procedure was to construct an LP model of a forest production system and use it to generate alternatives from which to test partial selections. A dynamically constrained model was utilized in the analysis exclusively.

## Constructing the LP Model

Data assembled by the Coconino National Forest staff in the current land management planning effort was used to build an LP model to investigate partial selections. The data consisted of production coefficients and costs by period for each prescription/analysis area combination, prices for valued outputs, and utilization coefficients for certain constraint capacities (for example, ending inventory coefficients). The data base was abridged from the 51 analysis areas that were identified by the forest staff on a forest-wide basis to 24 analysis areas representing commercial timber stands, i.e., stands capable of producing 20 cubic feet per acre per year. The planning horizon was shortened from 200 to 150 years, represented by decades for the first 50 years and by two 50-year periods for the remaining 100 years. The number of valued outputs carried in

the model was decreased to only two products. Trimming the model down was necessary to save on the cost of computer runs and is not considered to be a major drawback as the main trade-offs between timber and the other forest outputs are likely to occur on the commercial forest land. However, the model cannot be taken as an accurate representation of the Coconino National Forest planning model or the actual production capabilities of the entire forest.

In selecting products to include in the model, initial runs of the model were performed to determine if a given product had a range of production capabilities sufficient to provide a test of the hypotheses generated regarding partial selection of alternatives. Based on this analysis, sawtimber and grazing capacity were selected as the output variables for which partial selection output requirements were to be set. Water yield was also tested but could only be produced within very narrow limits, which was considered unsatisfactory for the purpose of this analysis. For convenience, water yield was left in the model as a valued output and appears in the solutions (Appendix A). However, output requirements for a given partial selection were set only for sawtimber and grazing capacity.

A fourth output included in the model was net timber yield, an aggregation of the yields of sawtimber, pulpwood and fuelwood. This output is not priced and was included in the model for the sole reason of imposing a nondeclining

even flow constraint. It was considered to be more appropriate to apply this constraint on the total output of all timber products rather than on sawtimber only.

The XMP linear programming package (Marsten 1981) was used in generating initial alternatives and testing partial selection alternatives. For this package, a user supplied program reads in the matrix coefficients and objective function values one column at a time and sets constraint levels and types. Upper and lower bounds can be specified for column variables. The basic structure of the LP model constructed for the production system being analyzed is shown in figure 11. The structure is similar to a Model I version FORPLAN (Johnson and Scheurman 1977).

Considering columns first, the first 27 represent the set of management prescriptions for analysis area one. Each prescription uses constraint capacity and generates outputs that are carried for each column as per acre production values for each time period in the planning horizon. There are a total of 587 prescriptions for the 24 analysis areas included in the model.

Columns 588 through 594 represent net timber production forest wide for each of the seven time periods. The following 21 columns, numbers 595 through 615, represent output by period for sawtimber, grazing capacity, and water yield. Each product requires one column for each period in the planning horizon.

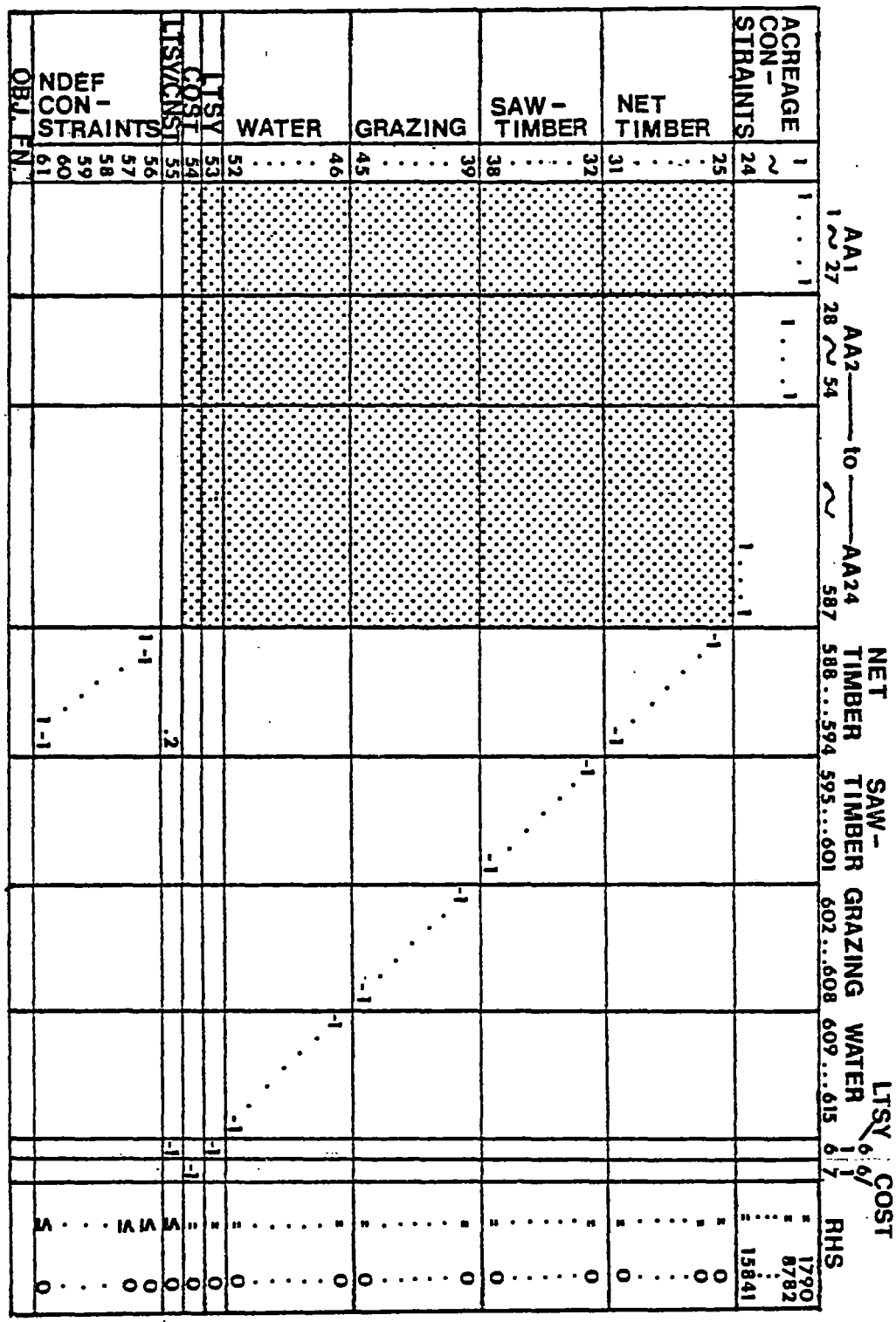


Figure 11. Matrix Structure for the 617 Column, 61 Row Linear Programming Model.

Column 616 is the long run sustained yield variable and 617 is total discounted cost over the planning horizon. The former was included in the model as a column variable in order to apply a constraint, the latter for placing bounds on budget and determining the cost of alternatives. The right hand side identifies the type ($\leq$, =, $\geq$) and level of capacity of the corresponding constraint (row).

The first 24 rows are acreage constraints designed to limit treatments to the available acreage in each analysis area. For example, row one simply sums the acreage allocated to the 27 available prescriptions for analysis area one and requires this sum be equal to 1790, the total acreage in analysis area one. The constraint is of the equality type because a less than or equal to formulation would allow no management at a cost of zero to be a possible prescription, which does not meet minimum legal requirements for soil, water, air and wildlife protection.

Rows 25 through 31 are accounting rows that contain the per acre output coefficients for net timber for 587 prescriptions and seven time periods. The negative one contained in each row under the corresponding column variable for net timber causes forest wide production of net timber by time period to flow into the column variables 588 through 594. For example, row 25 defines the mathematical relationship between the first 587 column variables, the set of prescriptions for the entire area being analyzed, and column

variable 588, the net timber produced in the first time period. Mathematically, each per acre output coefficient for net timber production in the first time period is multiplied by the acreage assigned to the corresponding prescription and the sum across all prescriptions and analysis areas is set equal to column variable 588, net timber produced in time period one.

Rows 32 through 52 set up the relationships for sawtimber, grazing capacity and water yield for each time period. Because the portion of the matrix containing these production coefficients had non-zero entries for almost every prescription, product and time period combination, it is called the dense portion of the matrix.

Row 53 contains coefficients for each prescription that represent the average per acre sustained yield capacity for a ten year period generated by the corresponding prescription. The resulting summation, when multiplied by the assigned acreages, is set equal to column variable number 616, total sustained yield capacity of the forest area being analyzed per ten year period.

Row 55 is the long run sustained yield constraint, requiring that one fifth times the final 50-year harvest be no greater than the sustained yield capacity generated by the corresponding land allocation on a per ten year period basis (the 0.2 coefficient for final period yield of net timber is to equate the length of the two time frames).

Row 54 defines the total cost, in present value terms, forest wide over the planning horizon. A single row is sufficient for cost (and long run sustained yield) because per period values are not considered necessary. All costs identified by the Coconino staff for treatments are included in the total cost of a given prescription and not just the costs to be incurred by the Forest Service.

The final row shown is the objective function. For a maximization of present net worth formulation it contains cost estimates for each prescription and prices for each output by period. Other forms of the objective function are of course possible. Prices can be dropped from the objective function for cost minimization problems or costs could be omitted for a maximization of present total benefits.

The XMP package also allows both upper and lower bounds to be placed on any column variable. For example, if an upper bound of $C_1$ is placed on the cost variable 617, then the budget cannot exceed this value. Lower bounds impose minimum values for column variables that must be achieved if the problem is to be feasible.

In making successive runs of the model on the computer, matrix coefficients, costs and constraint levels remain unchanged. Variations were made in the objective function, upper and lower bounds on column variables, and prices in the initial generation of alternatives from which the partial selections were made.

Optimizing the objective function subject to stated constraints and bounds will result in all 617 column variables being assigned values, although many will remain at zero levels. Values assigned to the first 587 variables determines the land allocation, the acreage devoted to each prescription. The remaining values identify the output levels of each product by period over the planning horizon, long run sustained yield capability of the forest for the given land allocation, and cost. In addition, information is provided regarding any nonequality constraints that are underachieved or surpassed in the solution. The final piece of data provided by the solution to the problem formulation is the value of the objective function.

## Generating a Set of Optimal Alternatives

Initially a set of optimal alternatives must be generated and described by LP runs in terms of output quantities and required budget levels for each alternative in the set. A maximization of present net worth using prices as given from the Coconino data was performed initially to generate the first alternative in the set. The budget level determined for this alternative was assumed to be one typically considered in the forest planning process.

Three additional alternatives with varying product emphases were located on the production possibilities curve for the present net worth maximization budget level. This was accomplished by placing an upper bound on cost at the

value of the PNW maximization budget level, and then maximizing present benefits for systematically varying relative prices for the two outputs, sawtimber and grazing capacity. Each LP run thus generates an optimal output combination for the given pair of relative prices and the allowable budget level (the PNW maximization budget level). It was determined in this preliminary analysis that relative price increases for sawtimber did not significantly alter the solution values obtained from the first alternative, PNW maximization.[6] Thus, the relative price of grazing capacity was systematically increased for successive LP runs, holding the price of sawtimber constant, to emphasize the production of grazing capacity relative to sawtimber. Of numerous relative price increases analyzed for grazing capacity, increases by factors of 3, 30, and 100 generated alternatives with significantly different product emphases. The set of four alternatives, corresponding to relative prices of grazing capacity increased by factors of 1, 3, 30 and 100 from the prices used by the Coconino National Forest, were assumed to represent adequately (for hypothesis testing) the range of product emphases possible for the forest at the PNW maximization budget level.

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[6]The PNW maximization generates a solution almost identical to a maximization of physical output of sawtimber. Thus, increasing the price for sawtimber had little or no affect on the solution.

The four alternatives correspond to points 1, 2, 3 and 4 in figure 12, a graphical illustration of the output combinations corresponding to the optimal alternatives forming the initial set. Partial selections of the four equal budget alternatives could generate alternatives with output combinations anywhere within the area bounded by line segments connecting the known points of optimality, 1, 2, 3 and 4.

A lower budget alternative, represented by point 5 in the figure, was generated for combining with the higher budget alternatives in testing partial selections of alternatives with different budget requirements. This was accomplished by setting an upper bound on cost at a value of one half the PNW maximization budget level, and maximizing present benefits using the original set of relative prices.

Returning to figure 12, partial selections between the set of five optimal alternatives can generate alternatives with output combinations anywhere within the area bounded by line segments connecting the known points of optimality, 1, 2, 3, 4 and 5.

Determining Partial Selections To Be Tested

The next step was to determine which alternatives to combine and at what fractional combination levels (values of s).

To examine partial selections of initial alternatives with different budget requirements, the lower budget

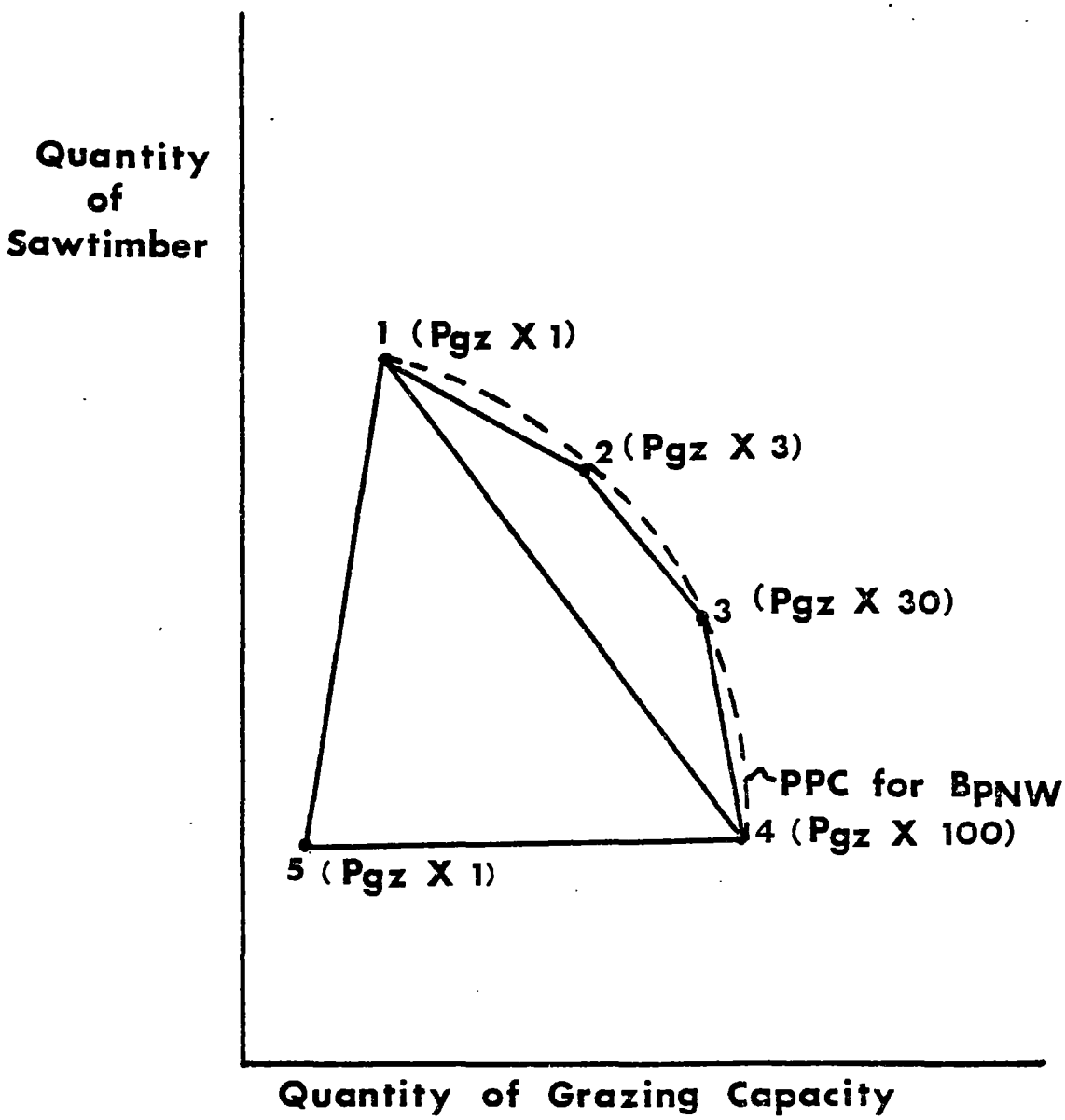


Figure 12. Possible Partial Selection Output Combinations for the Initial Set of Five Optimal Alternatives.

alternative was combined with each of the four higher budget alternatives. In an attempt to generate partial selections with the maximum error possible for the set of initial alternatives, 50-50 partial selections were tested. The maximum error is not guaranteed for equal selection of initial alternatives, but without prior knowledge of the functional form of marginal cost functions, there is no basis for predicting the value of s that maximizes error.

To determine if error is relatively low near initial (optimal) alternatives, as expected from the results of the theoretical considerations, 10-90 and 90-10 partial selections were also tested. Since the two fractional combination levels are equidistant in terms of production change from optimal alternatives for a given pair, symmetry of the total error function can be examined.

Combining the four pairs of initial alternatives at three fractional combination levels generates twelve different partial selection alternatives, or output combinations for which cost was determined by LP runs.

The number of partial selections that could be tested to determine the actual cost of partial selection output combinations was limited by the study budget. Thus, for testing partial selections of alternatives with the same budget requirement, only 50-50 partial selections were tested. In addition, only the alternative generated by maximizing PNW using the original relative prices was

combined with the other three alternatives with the same budget requirement. Combining the three resulting pairs of initial alternatives at the 50-50 level generated three partial selections to be tested. Thus a total of 15 partial selections were tested in the analysis, the output combinations for which are shown in figure 13. The labels on alternatives in the figure correspond to the run numbers in Table 2, a listing of all LP runs required in the analysis.

As shown in the table, five runs were required to generate the set of initial optimal alternatives. The remaining 15 runs were made to determine to actual cost for each of the partial selection output combinations.

In testing partial selections, required output levels are calculated from the original alternatives being combined on a per period basis. These per period production requirements for a given partial selection serve as lower bounds in a cost minimization LP run. Thus the minimum cost of producing the required partial selection output targets is given as the objective function value and can be compared to the associated predicted cost in determining the error, or degree of suboptimality of the partial selection alternative. The error determined for all tests can then be analyzed and related to the hypotheses developed from theoretical considerations.

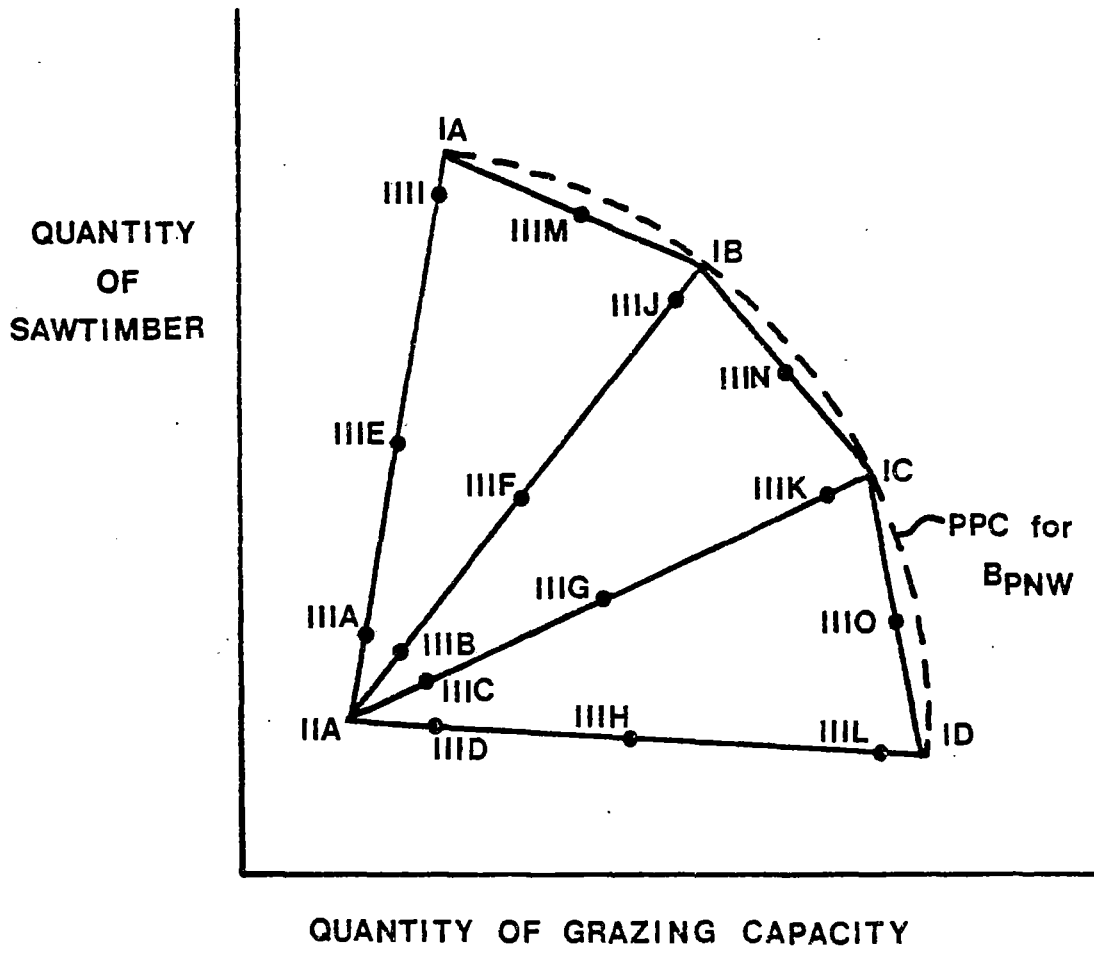


Figure 13. Required Output Combinations for the 15 Partial Selections To Be Tested.

Table 2. Description of linear programming runs to be performed in the analysis of partial selections.

| RUN SET | RUN # | RUN DESCRIPTION | DESCRIPTION OF BOUNDS | RUN OUTPUT |
|---|---|---|---|---|
| I | IA | PNW maximization best estimate prices | None | Budget level $B_1$ determined |
| | | 3 runs/budget level $B_1$ and following price emphases: | Budget $B_1$ | Three add'l alternatives on the PPC for |
| | IB | 3 X $P_{gz}$ | | $B_1$ determined |
| | IC | 30 X $P_{gz}$ | | |
| | ID | 100 X $P_{gz}$ | | |
| II | IIA | Budget level $B_1/2$, best estimate prices | Budget $B_1/2$ | Generates a low budget alternative |
| III | IIIA<br>IIIB<br>IIIC<br>IIID | 90-10 partial selections of IIA with each of the four higher budget alternative | Lower bounds set by period for partial selection output levels | Actual cost of producing the required targets determined |
| | IIIE<br>IIIF<br>IIIG<br>IIIH | 50-50 partial selections of IIA with each of the four higher budget alternatives | | |
| | IIII<br>IIIJ<br>IIIK<br>IIIL | 10-90 partial selections of IIA with each of the four higher budget alternatives | | |
| | IIIM<br>IIIN<br>IIIO | 50-50 partial selections of IA with IB, IC and ID | | |

# CHAPTER FOUR

## RESULTS

The initial step in generating the initial set of optimal alternatives was generation of the alternative which maximized PNW with the prices of the outputs set at those used by the Coconino National Forest planning staff. A total discounted cost of $210,557,600 was required for this alternative and served as an upper bound in generating the three additional alternatives, using relative price increases for grazing capacity by factors of 3, 30 and 100. The objective function for these three LP runs was maximization of present benefits.

For factors of 30 and 100, the allowable budget was not fully utilized (the upper bound was not binding). Cost for these two alternatives was $206,027,400 and $201,897,190 respectively. With an objective function of maximizing present benefits, production always has a positive net value, since costs are not included in the objective function. Thus it must be infeasible to increase the output level of either product from the output levels determined for the two alternatives without giving up a relatively larger value of the other product. This somewhat peculiar result was not considered to be a major problem. The impact on the

analysis is minimal. Since the four alternatives do not have equal budget requirements, the predicted cost must be calculated separately for each pair of alternatives being combined, and will vary slightly between pairs for a given partial selection proportion.

A lower budget alternative was generated using the original prices and an upper bound on cost of $105,278,880, half the budget level for the alternative which maximized PNW using the original prices, and again maximizing present benefits.

Total production and discounted costs over the planning horizon are listed in Table 3 for the set of five optimal alternatives, with a more detailed periodic listing being provided in Appendix A. As expected grazing capacity was substituted for sawtimber as the relative price of grazing capacity is increased.

Table 4 shows the required output quantities and predicted cost for each partial selection alternative to be tested. Each value was calculated as a weighted average, using the appropriate proportions, of the values for the initial alternatives shown in Table 3. It must be remembered that although only totals are shown, the required calculations were in actuality conducted on a per period basis. Per period output requirements can be derived from periodic output quantities in Appendix A.

Table 3. Total production and cost over the planning horizon for the initial set of optimal alternatives. $P_{gz}$ 2 represents the original price for grazing capacity used by the Coconino.

| RUN # | BUDGET | PRICE | EMP. | SAWTIMBER (MMBF) | GRAZING CAP'Y (MAUM) | COST |
|---|---|---|---|---|---|---|
| IA | HIGH | $P_{gz}$ | x I | 31,568 | 12,635 | 210,557,600 |
| IB | HIGH | $P_{gz}$ | x 3 | 30,035 | 31,619 | 210,557,600 |
| IC | HIGH | $P_{gz}$ | x 30 | 26,943 | 37,127 | 206,027,400 |
| ID | HIGH | $P_{gz}$ | x 100 | 19,516 | 40,550 | 201,897,190 |
| IIA | LOW | $P_{gz}$ | x I | 19,754 | 8,792 | 105,278,880 |

Table 4. Required total output quantities of sawtimber and grazing capacity and the predicted cost for each partial selection to be tested. Run numbers refer to Table 2.

| RUN # | ALTERNATIVES SELECTED | PS PROPOR-TION | SAWTIMBER (MMBF) | GRAZING CAP'Y (MAUM) | PREDICTED COST |
|---|---|---|---|---|---|
| IIIA | IA,IIA | 90-10 | 20,935 | 9,176 | 115,806,680 |
| IIIB | IB,IIA | 90-10 | 20,782 | 11,075 | 115,806,680 |
| IIIC | IC,IIA | 90-10 | 20,473 | 11,626 | 115,353,660 |
| IIID | ID,IIA | 90-10 | 19,730 | 11,968 | 114,940,640 |
| IIIE | IA,IIA | 50-50 | 25,661 | 10,714 | 157,918,200 |
| IIIF | IB,IIA | 50-50 | 24,895 | 20,206 | 157,918,200 |
| IIIG | IC,IIA | 50-50 | 23,349 | 22,960 | 155,653,100 |
| IIIH | ID,IIA | 50-50 | 19,635 | 24,671 | 153,588,000 |
| IIII | IA,IIA | 10-90 | 30,387 | 12,251 | 200,029,720 |
| IIIJ | IB,IIA | 10-90 | 29,007 | 29,336 | 200,029,720 |
| IIIK | IC,IIA | 10-90 | 26,224 | 34,294 | 195,952,540 |
| IIIL | ID,IIA | 10-90 | 19,540 | 37,374 | 192,235,350 |
| IIIM | IA,IB | 50-50 | 30,801 | 22,127 | 210,557,600 |
| IIIN | IA,IC | 50-50 | 29,256 | 24,881 | 208,292,500 |
| IIIO | IA,ID | 50-50 | 25,542 | 26,593 | 206,227,400 |

Table 5 shows the (minimum) actual cost of producing the required output quantities for each partial selection tested. For each test, actual cost is subtracted from the predicted cost to yield the magnitude of error, or degree of suboptimality, of the partial selection alternative. This value is then divided by the actual cost to get the percentage error of the alternatives. It is important to distinguish that the results of the theoretical discussion centered entirely on the magnitude of error in dollar terms, and not on percentages.

Hypothesis 1 proposed that partial selection alternatives will always be feasible but suboptimal for LP representations of forest production systems. For run IIIM, a feasible solution could not be generated. It is important to distinguish between the type of infeasibility encountered here and the type of infeasibility discussed in the theoretical analysis. In the earlier discussion it was stated that partial selection alternatives are infeasible if the actual cost of producing the required output levels exceeds the predicted cost. For run IIIM, the output combination cannot be produced <u>at any cost</u>. Thus there is more involved here than simply underestimating actual cost. Although the result obtained here could have occurred because of numerical rounding problems sometimes encountered using LP, it does cast some doubt as to the validity of the first hypothesis. The prior discussion may not have captured all

Table 5. Error and percentage error[a] determined for each partial selection tested. Run numbers refer to Table 2.

| RUN # | ALTERNATIVES SELECTED | PS PROPORTION | PREDICTED COST($) | ACTUAL COST($) | ERROR($) | % ERROR |
|---|---|---|---|---|---|---|
| IIIA | IA,IIA | 90-10 | 115,806,680 | 108,778,930 | 7,027,750 | 6.5 |
| IIIB | IB,IIA | 90-10 | 115,806,680 | 110,148,560 | 5,658,120 | 5.1 |
| IIIC | IC,IIA | 90-10 | 115,353,660 | 110,698,590 | 4,655,070 | 4.2 |
| IIID | ID,IIA | 90-10 | 114,940,640 | 110,410,710 | 4,529,930 | 4.1 |
| IIIE | IA,IIA | 50-50 | 157,918,200 | 126,547,360 | 31,370,840 | 24.8 |
| IIIF | IB,IIA | 50-50 | 157,918,200 | 131,738,580 | 26,179,620 | 19.9 |
| IIIG | IC,IIA | 50-50 | 155,653,100 | 133,764,320 | 21,888,780 | 16.4 |
| IIIH | ID,IIA | 50-50 | 153,588,000 | 133,422,180 | 20,165,820 | 15.1 |
| IIII | IA,IIA | 10-90 | 200,029,720 | 178,010,030 | 22,019,690 | 12.4 |
| IIIJ | IB,IIA | 10-90 | 200,029,720 | 180,583,170 | 19,446,550 | 10.8 |
| IIIK | IC,IIA | 10-90 | 195,952,540 | 180,694,450 | 15,258,090 | 8.4 |
| IIIL | ID,IIA | 10-90 | 192,235,350 | 174,160,080 | 18,075,270 | 10.4 |
| IIIM | IA,IB | 50-50 | 210,557,600 | INFEASIBLE | --------- | |
| IIIN | IA,IC | 50-50 | 208,292,500 | 186,765,400 | 21,527,100 | 11.5 |
| IIIO | IA,ID | 50-50 | 206,227,400 | 167,674,630 | 38,552,770 | 23.0 |

[a]Percentage error is calculated as predicted cost minus actual cost, divided by actual cost.

factors necessary to understand partial selections. However, the fact that no other tests generated infeasibilities (of either type) may suggest that the peculiar result obtained for run IIIM is not typical and should not lead to rejection of Hypothesis 1. At any rate, further investigation seems warranted to determine if infeasible partial selections are indeed a possibility in forest planning using LP.

For runs IIIN and IIIO, 50-50 combinations were taken of alternative IA with IC and with ID. The latter output combination generated is relatively more distant from alternative IA in terms of product emphasis. As expected from the previous discussion (figure 3), the more distant combination required by the second run (ID combined with IA) generates a larger magnitude of error due to the curvature of the production possibilities curve for the PNW maximization (high) budget level.

In examining partial selections involving alternatives with different budget requirements, it is more useful to group the tests by alternatives being combined, as in Table 6. One obvious result shown is that the error functions are not symmetrical, i.e., 10-90 and 90-10 partial selections do not generate the same magnitude of error. This result supports Hypothesis 3 for nonlinear marginal cost functions.

Table 6. Required output changes and error generated for partial selections grouped by alternatives being selected. Run numbers refer to Table 2.

| RUN # | PS PROP. | ALTERNATIVES COMBINED | % CHANGE IN PRODUCTION[b] IN SAWTIMBER | GRAZING CAPACITY | ERROR |
|---|---|---|---|---|---|
| IIIA | 90-10 | IA,IIA | 6 | 4 | 7,027,750 |
| IIIE | 50-50 | IA,IIA | 30 | 22 | 31,370,340 |
| IIII | 10-90 | IA,IIA | 54 | 39 | 22,019,690 |
| IIIB | 90-10 | IB,IIA | 5 | 26 | 5,658,120 |
| IIIF | 50-50 | IB,IIA | 26 | 130 | 26,179,620 |
| IIIJ | 10-90 | IB,IIA | 47 | 234 | 19,446,550 |
| IIIC | 90-10 | IC,IIA | 4 | 32 | 4,655,070 |
| IIIG | 50-50 | IC,IIA | 18 | 161 | 21,888,780 |
| IIIK | 10-90 | IC,IIA | 33 | 290 | 15,258,090 |
| IIID | 90-10 | ID,IIA | 0 | 36 | 4,529,930 |
| IIIH | 50-50 | ID,IIA | -1 | 181 | 20,165,820 |
| IIIL | 10-90 | ID,IIA | -1 | 325 | 18,075,270 |

[b]Percentage error is calculated as the required partial selection output quantity minus the quantity for the (optimal) lower budget alternative, divided by the lower budget alternative quantity.

For a given pair of alternatives, five error values were determined, two zero values for the error of the optimal alternatives in the pair (s = 0,1), and partial selection error values for three values of s (s = 0.1, 0.5 and 0.9). Figure 14 shows a typical representation of an error function for one of the alternative pairs. All four pairs of alternatives generated much larger error for 10-90 than for 90-10 partial selections, and the error for 50-50 selections was not much larger than for 10-90 selections. Thus, the illustration is representative of the functional form for partial selection error for any of the four pairs. The function is skewed toward the higher budget alternative.[7]

It is also possible to derive a general form of aggregate marginal cost function that would generate the representative error function indicated by the results. As shown in the figure, marginal cost must increase at an increasing rate. It seems intuitively reasonable that marginal cost will indeed increase more rapidly as a given output quantity is increased. The basis for this reasoning lies in proposing that the marginal productivity of inputs (treatments) becomes successively lower as production of a given output is increased.

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[7]In deriving the functional form for error, it is assumed that marginal cost is nondecreasing for any production change considered, i.e., Hypothesis 2 is assumed to hold.

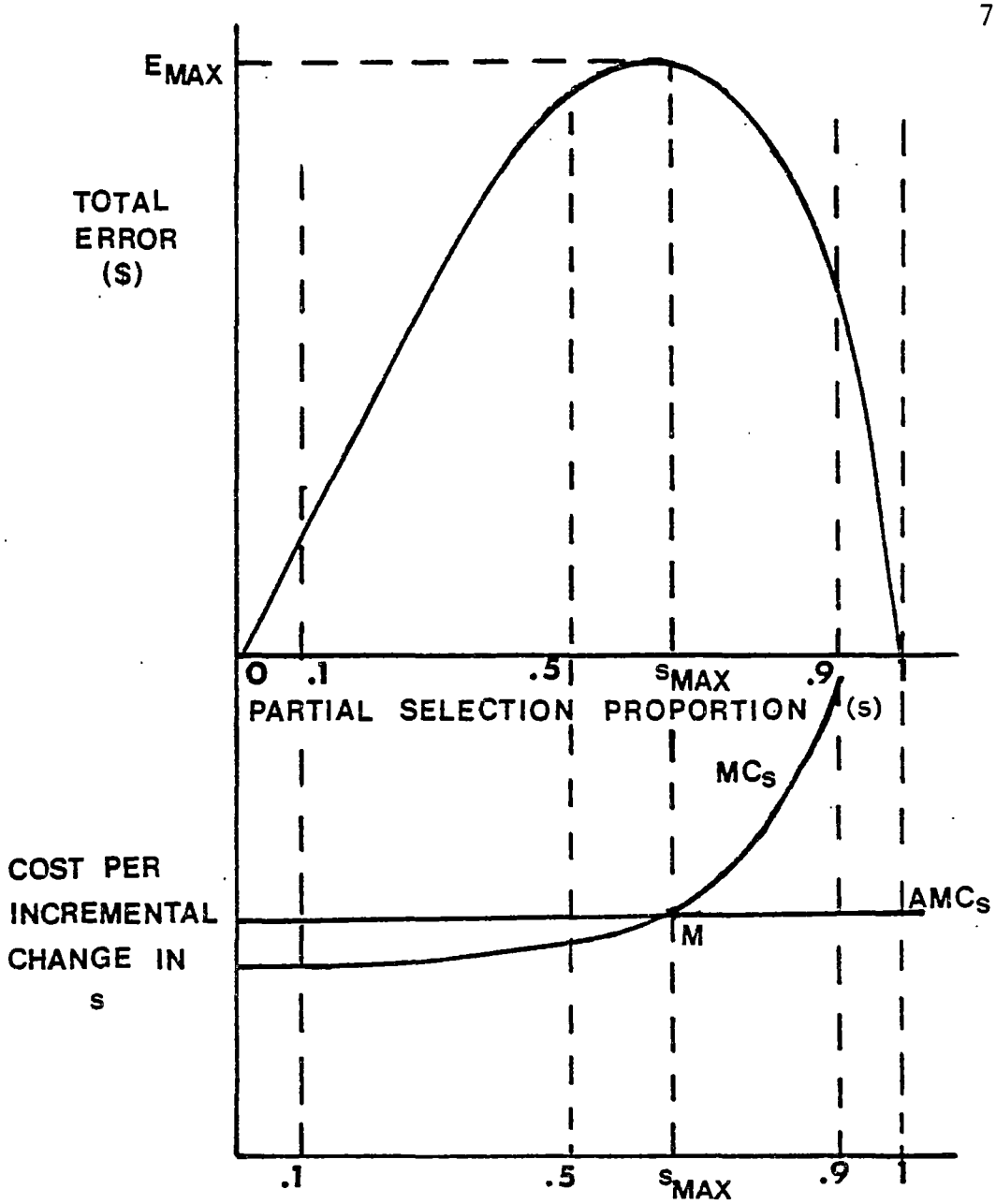


Figure 14. Representation of Proposed Functional Forms for Partial Selection Aggregate Error Function and Aggregate Marginal Cost Function.

Consider now the last three runs shown in Table 6, involving combinations of alternatives ID and the lower budget alternative IIA. As shown, sawtimber production remains essentially constant for the three values of s tested. For these three runs, the error tends to be lower, for given values of s, than for combinations of other alternatives. In fact, for a given partial selection proportion, error tends to increase for those runs resulting in successively higher levels of sawtimber production relative to grazing capacity. If independent marginal cost functions could be assumed, it could be reasoned that sawtimber generates a larger relative error for partial selections than does grazing capacity, i.e., the marginal cost function for sawtimber is a poorer approximation of a constant marginal cost function over the ranges of production considered here. However, if the marginal cost functions interact, that conclusion is not valid. Alternatively, it could be postulated that increasing the level of sawtimber production influences the marginal cost functional form for grazing capacity in such a way that it generates a larger error (the slope of the function now increases more rapidly). The two explanations proposed here coincide with two of the possible integration paths available in determining actual cost and therefore error.

# CHAPTER FIVE

## CONCLUSIONS

The most important result of this study may be that feasible partial selection alternatives are generated by an aggregate marginal cost function that is increasing. For analysis of the forest production system using LP, such an increasing function would be expected. This prediction, Hypothesis 1, was supported by the results but not proven[8]. Assuming Hypothesis 1 indeed holds, alternatives generated in forest planning that require partial selections will at least be feasible. Regional and national plans generated by a multilevel model that involve partial selections of individual forest alternatives will in actuality be operable, although there exists a cheaper (more efficient) way of producing the required output quantities (alternatives involving partial selections are suboptimal).

A suboptimal partial selection alternative may nonetheless be preferable to optimal alternatives. This is certainly the case when a multilevel model chooses a partial selection alternative over optimal alternatives. A decision

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[8]This conclusion is based on the observation that 14 of 15 partial selections tested were feasible. The single infeasibility is assumed to be due to numerical problems, and not a decreasing aggregate marginal cost function.

maker may also prefer a suboptimal partial selection alternative to the available optimal alternatives in the initial set. Therefore, it may be desirable to simply accept the error inherent in partial selections, particularly in situations where evaluation of additional alternatives cannot be carried out due to constraints on cost or time.

As an alternative to completely eliminating the entire evaluation process after the initial cycle of generating alternatives, only "preferred" partial selections would be evaluated. These correspond to partial selections chosen by a multilevel model or a partial selection alternative being chosen by a decision maker over the optimal alternatives in the initial set. Under this strategy, any partial selection of initial alternatives is permissible and is assumed to generate feasible alternatives, but only the preferred alternatives require evaluation after the initial cycle of generating optimal alternatives. Thus, explicit enumeration of alternatives is delayed until the actual choices are made. The partial selection alternatives will then be optimal once the evaluation process is complete.[9] This strategy may be useful in cases where time and cost do

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[9]For the case of multilevel models, the individual partial selection alternatives required at the forest level will be optimal once evaluated. However, including these new alternatives with adjusted cost values in the multilevel model formulation would not guarantee their total acceptance, i.e., regional and/or national alternatives will be suboptimal nonetheless.

not preclude evaluating some alternatives after the initial cycle.

When and how to use partial selections will depend on the nature and expense of the evaluation process. Any a priori knowledge of the functional form of the marginal cost functions involved can assist in determining if the cost of the evaluation process is warranted in light of the error generated for partial selections. This knowledge can be very general and still be useful. For example, if the functions increase at an increasing rate, as was the case for this analysis, a general form of error function can be predicted and used as a guide to estimate relative error for given values of s. The magnitude of error, however, cannot be predicted in such a manner without running initial tests to get representative values for the error of partial selections.

# APPENDIX A

## LP SOLUTION VALUES

The following computer printouts provide LP solution values for each run conducted in the analysis (run numbers refer to Table 2, page ). Solution values are provided for the following columns corresponding to the output variables included in the model (solution values for the individual prescriptions, columns 1-587, would be of little use to the reader and are omitted).

| Column # | Variable |
|---|---|
| 588 | Net timber (MCF) Period 1 |
| 589 | Net timber (MCF) Period 2 |
| . | . |
| . | . |
| . | . |
| 594 | Net timber (MCF) Period 7 |
| 595 | Sawtimber (MBF) Period 1 |
| . | . |
| . | . |
| . | . |
| 601 | Sawtimber (MBF) Period 7 |
| 602 | Grazing Cap. (AUM) Period 1 |
| . | . |
| . | . |
| . | . |
| 608 | Grazing Cap. (AUM) Period 7 |
| 609 | Water Yield (Ac.Ft.) Period 1 |
| . | . |
| . | . |
| . | . |
| 615 | Water Yield (Ac.Ft.) Period 7 |
| 616 | Long Term Sustained Yield (MCF) |
| 617 | Cost ($) |

Table A.1. RUN IA

Unfettered PNW Maximization

Using Best Estimate Prices

| Column # | | | Solution Value |
|---|---|---|---|
| 00200 | 588 | 28 | 0.32887813D+06 |
| 00300 | 589 | 27 | 0.32887803D+06 |
| 00400 | 590 | 35 | 0.32887793D+06 |
| 00500 | 591 | 56 | 0.32887783D+06 |
| 00600 | 592 | 33 | 0.32887773D+06 |
| 00700 | 593 | 55 | 0.16443881D+07 |
| 00800 | 594 | 49 | 0.16443880D+07 |
| 00900 | 595 | 31 | 0.16081433D+07 |
| 01000 | 596 | 13 | 0.10676641D+07 |
| 01100 | 597 | 29 | 0.14637801D+07 |
| 01200 | 598 | 39 | 0.12070940D+07 |
| 01300 | 599 | 60 | 0.13423271D+07 |
| 01400 | 600 | 50 | 0.16193312D+08 |
| 01500 | 601 | 52 | 0.86855199D+07 |
| 01600 | 602 | 54 | 0.76608605D+06 |
| 01700 | 603 | 17 | 0.57931023D+06 |
| 01800 | 604 | 26 | 0.54677215D+06 |
| 01900 | 605 | 34 | 0.53276627D+06 |
| 02000 | 606 | 30 | 0.57092768D+06 |
| 02100 | 607 | 41 | 0.30131259D+07 |
| 02200 | 608 | 37 | 0.66257548D+07 |
| 02300 | 609 | 40 | 0.25964243D+07 |
| 02400 | 610 | 32 | 0.24850855D+07 |
| 02500 | 611 | 43 | 0.24872896D+07 |
| 02600 | 612 | 47 | 0.24549683D+07 |
| 02700 | 613 | 44 | 0.25053304D+07 |
| 02800 | 614 | 45 | 0.12661026D+08 |
| 02900 | 615 | 51 | 0.14393067D+08 |
| 03000 | 616 | 46 | 0.32887751D+06 |
| 03100 | 617 | 38 | 0.21055760D+09 |
| 03200 | VALUE OF LINEAR OBJECTIVE FUNCTION= 0.12927274D+11 | | |

Table A.2. RUN IB

Maximization of Present Benefits Using a 3X Relative Price Increase for Grazing Capacity and Constraining Cost at or below the PNW Maximization Level

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.31984989D+06 |
| 589 | 27 | 0.31984979D+06 |
| 590 | 35 | 0.31984969D+06 |
| 591 | 56 | 0.31984959D+06 |
| 592 | 33 | 0.31984949D+06 |
| 593 | 55 | 0.15992469D+07 |
| 594 | 49 | 0.15992468D+07 |
| 595 | 31 | 0.15873384D+07 |
| 596 | 19 | 0.10942036D+07 |
| 597 | 29 | 0.12272033D+07 |
| 598 | 39 | 0.12472743D+07 |
| 599 | 60 | 0.13429129D+07 |
| 600 | 50 | 0.15883035D+08 |
| 601 | 52 | 0.76527168D+07 |
| 602 | 54 | 0.81066975D+06 |
| 603 | 17 | 0.79735094D+06 |
| 604 | 26 | 0.97432914D+06 |
| 605 | 34 | 0.12115310D+07 |
| 606 | 30 | 0.26339560D+07 |
| 607 | 41 | 0.10234251D+08 |
| 608 | 37 | 0.14956535D+08 |
| 609 | 40 | 0.26400773D+07 |
| 610 | 32 | 0.25000250D+07 |
| 611 | 43 | 0.25308899D+07 |
| 612 | 47 | 0.24880558D+07 |
| 613 | 44 | 0.25655601D+07 |
| 614 | 45 | 0.13114677D+08 |
| 615 | 51 | 0.14159920D+08 |
| 616 | 46 | 0.31984927D+06 |
| 617 | -1 | 0.21055760D+09 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.13699777D+11

Table A.3. RUN IC

Maximization of Present Benefits Using a 30X Relative Price Increase for Grazing Capacity and Constraining Cost at or below the PNW Maximization Level

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.28657274D+06 |
| 589 | 27 | 0.28657264D+06 |
| 590 | 35 | 0.28657254D+06 |
| 591 | 56 | 0.28657244D+06 |
| 592 | 33 | 0.28657234D+06 |
| 593 | 55 | 0.14328612D+07 |
| 594 | 49 | 0.14328611D+07 |
| 595 | 31 | 0.13047379D+07 |
| 596 | 4 | 0.12241005D+07 |
| 597 | 29 | 0.10028106D+07 |
| 598 | 39 | 0.10611245D+07 |
| 599 | 60 | 0.12837165D+07 |
| 600 | 50 | 0.14782510D+08 |
| 601 | 52 | 0.62844754D+07 |
| 602 | 54 | 0.84685202D+06 |
| 603 | 17 | 0.97375336D+06 |
| 604 | 26 | 0.13237100D+07 |
| 605 | 34 | 0.16872560D+07 |
| 606 | 30 | 0.33523444D+07 |
| 607 | 41 | 0.13239737D+08 |
| 608 | 37 | 0.15703065D+08 |
| 609 | 40 | 0.26665180D+07 |
| 610 | 32 | 0.25367956D+07 |
| 611 | 43 | 0.26265506D+07 |
| 612 | 47 | 0.25580201D+07 |
| 613 | 44 | 0.26606294D+07 |
| 614 | 45 | 0.13428079D+08 |
| 615 | 51 | 0.13627174D+08 |
| 616 | 46 | 0.28657212D+06 |
| 617 | 53 | 0.20602740D+09 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.27222270D+11

Table A.4. RUN ID

Maximization of Present Benefits Using a 100X Relative Price Increase for Grazing Capacity and Constraining Cost at or below the PNW Maximization Level

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.27037139D+06 |
| 589 | 27 | 0.27037129D+06 |
| 590 | 35 | 0.27037119D+06 |
| 591 | 56 | 0.27037109D+06 |
| 592 | 33 | 0.27037099D+06 |
| 593 | 55 | 0.13518544D+07 |
| 594 | 49 | 0.13518543D+07 |
| 595 | 31 | 0.11768134D+07 |
| 596 | 18 | 0.11659385D+07 |
| 597 | 29 | 0.95466641D+06 |
| 598 | 39 | 0.11460123D+07 |
| 599 | 60 | 0.12530196D+07 |
| 600 | 50 | 0.82792561D+07 |
| 601 | 52 | 0.55399020D+07 |
| 602 | 54 | 0.85423011D+06 |
| 603 | 17 | 0.11417440D+07 |
| 604 | 26 | 0.15797168D+07 |
| 605 | 34 | 0.19493465D+07 |
| 606 | 30 | 0.36900983D+07 |
| 607 | 41 | 0.14595365D+08 |
| 608 | 37 | 0.16739975D+08 |
| 609 | 40 | 0.26603294D+07 |
| 610 | 32 | 0.25413732D+07 |
| 611 | 43 | 0.26266923D+07 |
| 612 | 47 | 0.25839846D+07 |
| 613 | 44 | 0.26597681D+07 |
| 614 | 45 | 0.13551924D+08 |
| 615 | 51 | 0.13565122D+08 |
| 616 | 46 | 0.27037077D+06 |
| 617 | 38 | 0.20189719D+09 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.66643297D+11

Table A.5. RUN IIA

Maximization of Present Benefits Using Best Estimate Prices and Constraining Cost at or below Half the PNW Maximization Level

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.13101166D+06 |
| 589 | 27 | 0.13101156D+06 |
| 590 | 35 | 0.13101146D+06 |
| 591 | 56 | 0.17266552D+06 |
| 592 | 33 | 0.17266542D+06 |
| 593 | 55 | 0.10396823D+07 |
| 594 | 49 | 0.10771264D+07 |
| 595 | 31 | 0.53222058D+06 |
| 596 | 57 | 0.28318482D+06 |
| 597 | 29 | 0.57232699D+06 |
| 598 | 39 | 0.48480913D+06 |
| 599 | 60 | 0.73082534D+06 |
| 600 | 50 | 0.12988863D+08 |
| 601 | 52 | 0.41614142D+07 |
| 602 | 54 | 0.73475156D+06 |
| 603 | 17 | 0.54698197D+06 |
| 604 | 26 | 0.49344051D+06 |
| 605 | 34 | 0.47818272D+06 |
| 606 | 30 | 0.47843373D+06 |
| 607 | 41 | 0.25892278D+07 |
| 608 | 37 | 0.34708211D+07 |
| 609 | 40 | 0.25751180D+07 |
| 610 | 32 | 0.24612228D+07 |
| 611 | 43 | 0.24094938D+07 |
| 612 | 47 | 0.23285057D+07 |
| 613 | 44 | 0.23217590D+07 |
| 614 | 45 | 0.11721115D+08 |
| 615 | 51 | 0.12300379D+08 |
| 616 | 46 | 0.21542519D+06 |
| 617 | -1 | 0.10527880D+09 |
| 675 | 21 | 0.41654156D+05 |
| 677 | 11 | 0.35271138D+05 |
| 678 | 6 | 0.37444251D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.88532970D+10

Table A.6. RUN IIIA

90-10 Partial Selection of IIA with IA,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.14707835D+06 |
| 589 | 27 | 0.14707825D+06 |
| 590 | 58 | 0.14707815D+06 |
| 591 | 56 | 0.18583549D+06 |
| 592 | 60 | 0.18583539D+06 |
| 593 | 50 | 0.10970830D+07 |
| 594 | 25 | 0.11648268D+07 |
| 595 | 0 | 0.63981285D+06 |
| 596 | 0 | 0.36163275D+06 |
| 597 | 0 | 0.66147230D+06 |
| 598 | 0 | 0.55703762D+06 |
| 599 | 23 | 0.79870704D+06 |
| 600 | 0 | 0.13309308D+08 |
| 601 | 0 | 0.46138248D+07 |
| 602 | 0 | 0.73788501D+06 |
| 603 | 18 | 0.55459459D+06 |
| 604 | 20 | 0.50160034D+06 |
| 605 | 4 | 0.49222203D+06 |
| 606 | 34 | 0.51881153D+06 |
| 607 | 26 | 0.27834565D+07 |
| 608 | 0 | 0.37863145D+07 |
| 609 | 47 | 0.25816056D+07 |
| 610 | 48 | 0.24707550D+07 |
| 611 | 30 | 0.24166501D+07 |
| 612 | 61 | 0.23400323D+07 |
| 613 | 49 | 0.23322085D+07 |
| 614 | 46 | 0.11818934D+08 |
| 615 | 51 | 0.12443896D+08 |
| 616 | 12 | 0.23296526D+06 |
| 617 | 52 | 0.10877893D+09 |
| 675 | 13 | 0.38757434D+05 |
| 677 | 31 | 0.33581323D+05 |
| 678 | 16 | 0.67743882D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.10877893D+09

Table A.7. RUN IIIB

90-10 Partial Selection of IIA with IB,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.14493981D+06 |
| 589 | 27 | 0.14493971D+06 |
| 590 | 58 | 0.14493961D+06 |
| 591 | 56 | 0.18606758D+06 |
| 592 | 60 | 0.18606748D+06 |
| 593 | 50 | 0.10923720D+07 |
| 594 | 25 | 0.11457954D+07 |
| 595 | 0 | 0.63773236D+06 |
| 596 | 0 | 0.36428670D+06 |
| 597 | 0 | 0.63781462D+06 |
| 598 | 0 | 0.56105565D+06 |
| 599 | 0 | 0.79203409D+06 |
| 600 | 0 | 0.13278280D+08 |
| 601 | 0 | 0.45105444D+07 |
| 602 | 0 | 0.74234338D+06 |
| 603 | 18 | 0.57441362D+06 |
| 604 | 1 | 0.54171468D+06 |
| 605 | 0 | 0.55151755D+06 |
| 606 | 0 | 0.69398595D+06 |
| 607 | 41 | 0.34767094D+07 |
| 608 | 0 | 0.46193925D+07 |
| 609 | 47 | 0.25822822D+07 |
| 610 | 48 | 0.24696450D+07 |
| 611 | 30 | 0.24189592D+07 |
| 612 | 61 | 0.23413905D+07 |
| 613 | 49 | 0.23360078D+07 |
| 614 | 46 | 0.11831776D+08 |
| 615 | 51 | 0.12427011D+08 |
| 616 | 12 | 0.22915899D+06 |
| 617 | 52 | 0.11014856D+09 |
| 675 | 16 | 0.41128070D+05 |
| 677 | 45 | 0.32407027D+05 |
| 678 | 36 | 0.53423496D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.11014856D+09

Table A.8. RUN IIIC

90-10 Partial Selection of IIA with IC,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.15095651D+06 |
| 589 | 27 | 0.15095641D+06 |
| 590 | 58 | 0.15095631D+06 |
| 591 | 56 | 0.18138953D+06 |
| 592 | 60 | 0.18138943D+06 |
| 593 | 50 | 0.10864097D+07 |
| 594 | 25 | 0.11311377D+07 |
| 595 | 0 | 0.60947231D+06 |
| 596 | 0 | 0.37727639D+06 |
| 597 | 42 | 0.63999851D+06 |
| 598 | 0 | 0.54244066D+06 |
| 599 | 0 | 0.78611445D+06 |
| 600 | 0 | 0.13168228D+08 |
| 601 | 0 | 0.43737203D+07 |
| 602 | 0 | 0.74596161D+06 |
| 603 | 20 | 0.58977096D+06 |
| 604 | 0 | 0.57646746D+06 |
| 605 | 0 | 0.59909005D+06 |
| 606 | 39 | 0.82154352D+06 |
| 607 | 34 | 0.39453726D+07 |
| 608 | 40 | 0.51241465D+07 |
| 609 | 47 | 0.25833904D+07 |
| 610 | 48 | 0.24733216D+07 |
| 611 | 30 | 0.24290837D+07 |
| 612 | 61 | 0.23581643D+07 |
| 613 | 49 | 0.23523870D+07 |
| 614 | 46 | 0.11862325D+08 |
| 615 | 51 | 0.12402048D+08 |
| 616 | 12 | 0.22622743D+06 |
| 617 | 52 | 0.11069859D+09 |
| 675 | 3 | 0.30433322D+05 |
| 677 | 32 | 0.35892610D+05 |
| 678 | 37 | 0.44728054D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.11069859D+09

Table A.9. RUN IIID

90-10 Partial Selection of IIA with ID,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.14884716D+06 |
| 589 | 27 | 0.14884706D+06 |
| 590 | 58 | 0.14884696D+06 |
| 591 | 56 | 0.17447481D+06 |
| 592 | 60 | 0.17447471D+06 |
| 593 | 50 | 0.10305737D+07 |
| 594 | 25 | 0.11105718D+07 |
| 595 | 0 | 0.59667986D+06 |
| 596 | 0 | 0.37146019D+06 |
| 597 | 0 | 0.61056093D+06 |
| 598 | 0 | 0.55092945D+06 |
| 599 | 0 | 0.78304477D+06 |
| 600 | 0 | 0.12517902D+08 |
| 601 | 0 | 0.42992630D+07 |
| 602 | 0 | 0.74669941D+06 |
| 603 | 0 | 0.60645817D+06 |
| 604 | 0 | 0.60206814D+06 |
| 605 | 0 | 0.62529910D+06 |
| 606 | 10 | 0.87988692D+06 |
| 607 | 29 | 0.40814795D+07 |
| 608 | 8 | 0.52451983D+07 |
| 609 | 47 | 0.25825272D+07 |
| 610 | 48 | 0.24731706D+07 |
| 611 | 30 | 0.24271997D+07 |
| 612 | 61 | 0.23591353D+07 |
| 613 | 49 | 0.23522723D+07 |
| 614 | 46 | 0.11843125D+08 |
| 615 | 51 | 0.12304082D+08 |
| 616 | 12 | 0.22211427D+06 |
| 617 | 52 | 0.11041071D+09 |
| 675 | 42 | 0.25627957D+05 |
| 677 | 9 | 0.31640132D+05 |
| 678 | 2 | 0.79998215D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.11041071D+09

Table A.10. RUN IIIE

50-50 Partial Selection of IIA with IA,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.24815771D+06 |
| 589 | 27 | 0.24815761D+06 |
| 590 | 58 | 0.24815751D+06 |
| 591 | 56 | 0.24898949D+06 |
| 592 | 60 | 0.27376504D+06 |
| 593 | 50 | 0.14761425D+07 |
| 594 | 25 | 0.14761424D+07 |
| 595 | 0 | 0.10701819D+07 |
| 596 | 0 | 0.67542446D+06 |
| 597 | 0 | 0.10180535D+07 |
| 598 | 0 | 0.84595156D+06 |
| 599 | 0 | 0.10365762D+07 |
| 600 | 0 | 0.14591088D+08 |
| 601 | 0 | 0.64234671D+07 |
| 602 | 0 | 0.75041880D+06 |
| 603 | 44 | 0.58748638D+06 |
| 604 | 10 | 0.54109002D+06 |
| 605 | 32 | 0.54373490D+06 |
| 606 | 36 | 0.62450036D+06 |
| 607 | 59 | 0.31715236D+07 |
| 608 | 0 | 0.50482879D+07 |
| 609 | 47 | 0.25899668D+07 |
| 610 | 48 | 0.25132134D+07 |
| 611 | 30 | 0.24689541D+07 |
| 612 | 61 | 0.24287704D+07 |
| 613 | 49 | 0.24066867D+07 |
| 614 | 46 | 0.12249985D+08 |
| 615 | 51 | 0.13303977D+08 |
| 616 | 12 | 0.29522837D+06 |
| 617 | 52 | 0.12654736D+09 |
| 675 | 54 | 0.83207980D+03 |
| 676 | 26 | 0.24775653D+05 |
| 677 | 42 | 0.21463550D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.12654736D+09

Table A.11. RUN IIIF

50-50 Partial Selection of IIA with IB,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.24499866D+06 |
| 589 | 27 | 0.24499856D+06 |
| 590 | 58 | 0.24499846D+06 |
| 591 | 56 | 0.25880861D+06 |
| 592 | 60 | 0.25880851D+06 |
| 593 | 50 | 0.14254236D+07 |
| 594 | 25 | 0.14481800D+07 |
| 595 | 0 | 0.10597795D+07 |
| 596 | 0 | 0.68869421D+06 |
| 597 | 16 | 0.94150310D+06 |
| 598 | 42 | 0.89846313D+06 |
| 599 | 0 | 0.10368691D+07 |
| 600 | 0 | 0.14435949D+08 |
| 601 | 0 | 0.59070655D+07 |
| 602 | 0 | 0.77271066D+06 |
| 603 | 21 | 0.68734458D+06 |
| 604 | 2 | 0.74713042D+06 |
| 605 | 1 | 0.86767890D+06 |
| 606 | 18 | 0.16243775D+07 |
| 607 | 0 | 0.64117394D+07 |
| 608 | 0 | 0.92136780D+07 |
| 609 | 47 | 0.26010684D+07 |
| 610 | 48 | 0.25181099D+07 |
| 611 | 30 | 0.24850956D+07 |
| 612 | 61 | 0.24521272D+07 |
| 613 | 49 | 0.24487522D+07 |
| 614 | 46 | 0.12445141D+08 |
| 615 | 51 | 0.13203170D+08 |
| 616 | 12 | 0.28963590D+06 |
| 617 | 52 | 0.13173858D+09 |
| 675 | 11 | 0.13810248D+05 |
| 677 | 35 | 0.26276319D+05 |
| 678 | 7 | 0.22756471D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.13173858D+09

Table A.12. RUN IIIG

50-50 Partial Selection of IIA with IC,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.22732742D+06 |
| 589 | 27 | 0.22732732D+06 |
| 590 | 58 | 0.22732722D+06 |
| 591 | 56 | 0.22732712D+06 |
| 592 | 60 | 0.22732702D+06 |
| 593 | 50 | 0.12872349D+07 |
| 594 | 25 | 0.13980688D+07 |
| 595 | 0 | 0.91847924D+06 |
| 596 | 0 | 0.75364266D+06 |
| 597 | 0 | 0.78756880D+06 |
| 598 | 6 | 0.80132652D+06 |
| 599 | 0 | 0.10072709D+07 |
| 600 | 0 | 0.13885686D+08 |
| 601 | 0 | 0.52229448D+07 |
| 602 | 0 | 0.79080179D+06 |
| 603 | 39 | 0.77296125D+06 |
| 604 | 0 | 0.90857526D+06 |
| 605 | 0 | 0.10827194D+07 |
| 606 | 43 | 0.22072269D+07 |
| 607 | 38 | 0.85245745D+07 |
| 608 | 19 | 0.11374442D+08 |
| 609 | 47 | 0.26185500D+07 |
| 610 | 48 | 0.25180933D+07 |
| 611 | 30 | 0.25017147D+07 |
| 612 | 61 | 0.24567334D+07 |
| 613 | 49 | 0.24688246D+07 |
| 614 | 46 | 0.12511997D+08 |
| 615 | 51 | 0.12913729D+08 |
| 616 | 12 | 0.27961367D+06 |
| 617 | 52 | 0.13376432D+09 |
| 677 | 26 | 0.30120072D+05 |
| 678 | 42 | 0.11083399D+06 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.13376432D+09

Table A.13. RUN IIIH

50-50 Partial Selection of IIA with ID,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.21459524D+06 |
| 589 | 27 | 0.21459514D+06 |
| 590 | 58 | 0.21459504D+06 |
| 591 | 56 | 0.21459494D+06 |
| 592 | 60 | 0.21459484D+06 |
| 593 | 50 | 0.11290626D+07 |
| 594 | 25 | 0.12466860D+07 |
| 595 | 0 | 0.85451699D+06 |
| 596 | 0 | 0.72456166D+06 |
| 597 | 0 | 0.76349670D+06 |
| 598 | 41 | 0.83833096D+06 |
| 599 | 0 | 0.99192247D+06 |
| 600 | 0 | 0.10634060D+08 |
| 601 | 0 | 0.48506581D+07 |
| 602 | 0 | 0.79449084D+06 |
| 603 | 15 | 0.85708226D+06 |
| 604 | 0 | 0.10365787D+07 |
| 605 | 0 | 0.12137646D+07 |
| 606 | 23 | 0.25389047D+07 |
| 607 | 13 | 0.92935305D+07 |
| 608 | 20 | 0.12517152D+08 |
| 609 | 47 | 0.26123726D+07 |
| 610 | 48 | 0.25151371D+07 |
| 611 | 30 | 0.25049053D+07 |
| 612 | 61 | 0.24697216D+07 |
| 613 | 49 | 0.24875281D+07 |
| 614 | 46 | 0.12507366D+08 |
| 615 | 51 | 0.12699719D+08 |
| 616 | 12 | 0.24933709D+06 |
| 617 | 52 | 0.13342218D+09 |
| 677 | 57 | 0.11217775D+05 |
| 678 | 5 | 0.11762351D+06 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.13342218D+09

Table A.14. RUN IIII

10-90 Partial Selection of IIA with IA,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.31779009D+06 |
| 589 | 27 | 0.31778999D+06 |
| 590 | 58 | 0.31778989D+06 |
| 591 | 56 | 0.32216783D+06 |
| 592 | 60 | 0.32216773D+06 |
| 593 | 50 | 0.16108382D+07 |
| 594 | 25 | 0.16108381D+07 |
| 595 | 5 | 0.15210722D+07 |
| 596 | 0 | 0.98921617D+06 |
| 597 | 0 | 0.13746348D+07 |
| 598 | 2 | 0.11817704D+07 |
| 599 | 0 | 0.12811769D+07 |
| 600 | 0 | 0.15872867D+08 |
| 601 | 0 | 0.82331093D+07 |
| 602 | 0 | 0.76295260D+06 |
| 603 | 55 | 0.57954127D+06 |
| 604 | 59 | 0.54699342D+06 |
| 605 | 14 | 0.54794282D+06 |
| 606 | 32 | 0.62322262D+06 |
| 607 | 16 | 0.36065903D+07 |
| 608 | 0 | 0.63102614D+07 |
| 609 | 47 | 0.25964093D+07 |
| 610 | 48 | 0.24946755D+07 |
| 611 | 30 | 0.24813680D+07 |
| 612 | 61 | 0.24448239D+07 |
| 613 | 49 | 0.24853662D+07 |
| 614 | 46 | 0.12660500D+08 |
| 615 | 51 | 0.14033126D+08 |
| 616 | 12 | 0.32216751D+06 |
| 617 | 52 | 0.17801003D+09 |
| 675 | 11 | 0.43780469D+04 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.17801003D+09

Table A.15. RUN IIIJ

10-90 Partial Selection of IIA with IB,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.31156990D+06 |
| 589 | 27 | 0.31156980D+06 |
| 590 | 58 | 0.31156970D+06 |
| 591 | 56 | 0.31156960D+06 |
| 592 | 60 | 0.31156950D+06 |
| 593 | 50 | 0.15640510D+07 |
| 594 | 25 | 0.15640509D+07 |
| 595 | 0 | 0.14818266D+07 |
| 596 | 0 | 0.10131017D+07 |
| 597 | 43 | 0.12168014D+07 |
| 598 | 31 | 0.11737383D+07 |
| 599 | 1 | 0.12819411D+07 |
| 600 | 0 | 0.15593618D+08 |
| 601 | 0 | 0.73035866D+07 |
| 602 | 0 | 0.80307793D+06 |
| 603 | 0 | 0.77231405D+06 |
| 604 | 0 | 0.92624027D+06 |
| 605 | 38 | 0.11450966D+07 |
| 606 | 35 | 0.24626219D+07 |
| 607 | 0 | 0.94697486D+07 |
| 608 | 0 | 0.13807964D+08 |
| 609 | 47 | 0.26250541D+07 |
| 610 | 48 | 0.25132030D+07 |
| 611 | 30 | 0.25220113D+07 |
| 612 | 61 | 0.24777260D+07 |
| 613 | 49 | 0.25441810D+07 |
| 614 | 46 | 0.12954871D+08 |
| 615 | 51 | 0.13846724D+08 |
| 616 | 12 | 0.31281007D+06 |
| 617 | 52 | 0.18058317D+09 |
| 677 | 4 | 0.12407924D+04 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.18058317D+09

Table A.16. RUN IIIK

10-90 Partial Selection of IIA with IC,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.28030209D+06 |
| 589 | 27 | 0.28030199D+06 |
| 590 | 58 | 0.28030189D+06 |
| 591 | 56 | 0.28030179D+06 |
| 592 | 60 | 0.28030169D+06 |
| 593 | 50 | 0.14099946D+07 |
| 594 | 25 | 0.14255620D+07 |
| 595 | 0 | 0.12274862D+07 |
| 596 | 0 | 0.11300089D+07 |
| 597 | 37 | 0.97643679D+06 |
| 598 | 43 | 0.10157578D+07 |
| 599 | 42 | 0.12408746D+07 |
| 600 | 0 | 0.14603145D+08 |
| 601 | 0 | 0.60721693D+07 |
| 602 | 0 | 0.83564198D+06 |
| 603 | 0 | 0.93107622D+06 |
| 604 | 0 | 0.12406830D+07 |
| 605 | 0 | 0.15663487D+07 |
| 606 | 36 | 0.31510319D+07 |
| 607 | 0 | 0.12174686D+08 |
| 608 | 31 | 0.15077066D+08 |
| 609 | 47 | 0.26669001D+07 |
| 610 | 48 | 0.25379802D+07 |
| 611 | 30 | 0.26005584D+07 |
| 612 | 61 | 0.25373268D+07 |
| 613 | 49 | 0.26229272D+07 |
| 614 | 46 | 0.13221666D+08 |
| 615 | 51 | 0.13499391D+08 |
| 616 | 12 | 0.28511230D+06 |
| 617 | 52 | 0.18069445D+09 |
| 677 | 39 | 0.16973241D+04 |
| 678 | 7 | 0.15567511D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.18069445D+09

Table A.17. RUN IIIL

10-90 Partial Selection of IIA with ID,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.27485395D+06 |
| 589 | 27 | 0.27485385D+06 |
| 590 | 58 | 0.27485375D+06 |
| 591 | 56 | 0.27485365D+06 |
| 592 | 60 | 0.27485355D+06 |
| 593 | 50 | 0.13742672D+07 |
| 594 | 25 | 0.13742671D+07 |
| 595 | 0 | 0.11123541D+07 |
| 596 | 0 | 0.10776631D+07 |
| 597 | 44 | 0.96977573D+06 |
| 598 | 9 | 0.11403091D+07 |
| 599 | 24 | 0.12996200D+07 |
| 600 | 0 | 0.87502167D+07 |
| 601 | 0 | 0.54020533D+07 |
| 602 | 0 | 0.84228226D+06 |
| 603 | 26 | 0.10899221D+07 |
| 604 | 0 | 0.14710892D+07 |
| 605 | 0 | 0.18022301D+07 |
| 606 | 41 | 0.34959089D+07 |
| 607 | 0 | 0.13394751D+08 |
| 608 | 29 | 0.15838832D+08 |
| 609 | 47 | 0.26689948D+07 |
| 610 | 48 | 0.25465143D+07 |
| 611 | 30 | 0.26167500D+07 |
| 612 | 61 | 0.25815261D+07 |
| 613 | 49 | 0.26576486D+07 |
| 614 | 46 | 0.13382160D+08 |
| 615 | 51 | 0.13473305D+08 |
| 616 | 12 | 0.27485333D+06 |
| 617 | 52 | 0.17416008D+09 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.17416008D+09

Table A.18. RUN IIIM

50-50 Partial Selection of IA with IB,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.32282374D+06 |
| 589 | 27 | 0.32282364D+06 |
| 590 | 58 | 0.32282354D+06 |
| 591 | 56 | 0.32282344D+06 |
| 592 | 60 | 0.32282334D+06 |
| 593 | 50 | 0.16141162D+07 |
| 594 | 25 | 0.16141161D+07 |
| 595 | 0 | 0.15977408D+07 |
| 596 | 0 | 0.10809338D+07 |
| 597 | 0 | 0.13454917D+07 |
| 598 | 0 | 0.12271842D+07 |
| 599 | 0 | 0.13426200D+07 |
| 600 | 0 | 0.16038174D+08 |
| 601 | 0 | 0.81691184D+07 |
| 602 | 0 | 0.78837790D+06 |
| 603 | 54 | 0.69075348D+06 |
| 604 | 42 | 0.76505211D+06 |
| 605 | 5 | 0.87421663D+06 |
| 606 | 0 | 0.16024418D+07 |
| 607 | 0 | 0.66236884D+07 |
| 608 | 22 | 0.10847692D+08 |
| 609 | 47 | 0.26231486D+07 |
| 610 | 48 | 0.24918638D+07 |
| 611 | 30 | 0.25100510D+07 |
| 612 | 61 | 0.24754395D+07 |
| 613 | 49 | 0.25403789D+07 |
| 614 | 46 | 0.12871312D+08 |
| 615 | 51 | 0.14261691D+08 |
| 616 | 12 | 0.32282312D+06 |
| 617 | 52 | 0.21435039D+09 |
| 654 | -3 | 0.74272538D+05 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.74272538D+05

Table A.19. RUN IIIN

50-50 Partial Selection of IA with IC,

Objective of Minimizing Cost of Producing

the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.31343636D+06 |
| 589 | 27 | 0.31343626D+06 |
| 590 | 58 | 0.31343616D+06 |
| 591 | 56 | 0.31343606D+06 |
| 592 | 60 | 0.31343596D+06 |
| 593 | 50 | 0.15694212D+07 |
| 594 | 25 | 0.15694211D+07 |
| 595 | 0 | 0.14564406D+07 |
| 596 | 0 | 0.11458823D+07 |
| 597 | 37 | 0.12592965D+07 |
| 598 | 9 | 0.11771254D+07 |
| 599 | 0 | 0.13130218D+07 |
| 600 | 0 | 0.15487911D+08 |
| 601 | 0 | 0.74849976D+07 |
| 602 | 0 | 0.80646903D+06 |
| 603 | 7 | 0.78299710D+06 |
| 604 | 0 | 0.93524108D+06 |
| 605 | 24 | 0.11139619D+07 |
| 606 | 15 | 0.23418657D+07 |
| 607 | 45 | 0.89387145D+07 |
| 608 | 54 | 0.13193426D+08 |
| 609 | 47 | 0.26271425D+07 |
| 610 | 48 | 0.25159503D+07 |
| 611 | 30 | 0.25352908D+07 |
| 612 | 61 | 0.24839324D+07 |
| 613 | 49 | 0.25580757D+07 |
| 614 | 46 | 0.12942307D+08 |
| 615 | 51 | 0.13883960D+08 |
| 616 | 12 | 0.31388412D+06 |
| 617 | 52 | 0.18676540D+09 |
| 677 | 8 | 0.44838177D+03 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.18676540D+09

Table A.20. RUN IIIO

50-50 Partial Selection of IA with ID,
Objective of Minimizing Cost of Producing
the Required Output Quantities

| Column # | | Solution Value |
|---|---|---|
| 588 | 28 | 0.31252322D+06 |
| 589 | 27 | 0.31252312D+06 |
| 590 | 58 | 0.31252302D+06 |
| 591 | 56 | 0.31252292D+06 |
| 592 | 60 | 0.31252282D+06 |
| 593 | 50 | 0.15626136D+07 |
| 594 | 25 | 0.15626135D+07 |
| 595 | 0 | 0.13924783D+07 |
| 596 | 0 | 0.11168013D+07 |
| 597 | 0 | 0.12092233D+07 |
| 598 | 9 | 0.13192415D+07 |
| 599 | 0 | 0.12976733D+07 |
| 600 | 0 | 0.12236284D+08 |
| 601 | 0 | 0.71127109D+07 |
| 602 | 0 | 0.81015808D+06 |
| 603 | 0 | 0.86052712D+06 |
| 604 | 0 | 0.10632445D+07 |
| 605 | 0 | 0.12410564D+07 |
| 606 | 45 | 0.24580365D+07 |
| 607 | 20 | 0.88299112D+07 |
| 608 | 15 | 0.13155614D+08 |
| 609 | 47 | 0.26430778D+07 |
| 610 | 48 | 0.25292037D+07 |
| 611 | 30 | 0.25669926D+07 |
| 612 | 61 | 0.25533235D+07 |
| 613 | 49 | 0.26242123D+07 |
| 614 | 46 | 0.12811193D+08 |
| 615 | 51 | 0.13819163D+08 |
| 616 | 12 | 0.31252260D+06 |
| 617 | 52 | 0.16767463D+09 |

VALUE OF LINEAR OBJECTIVE FUNCTION= 0.16767463D+09

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C.1

I-33 Bookplate
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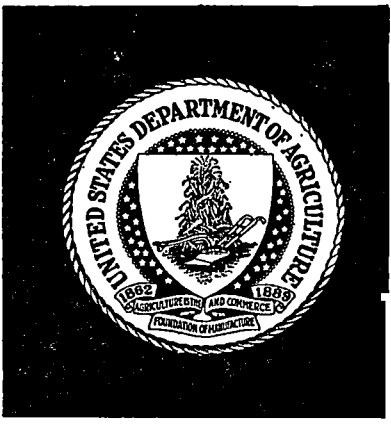


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